6.

THE ART OF ARITHMETICKE IN WHOLE NVMBERS AND FRACTIONS.

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In a more readie and easie method then hitherto hath bene published. Written in Latin by

P. RAMVS:

la Ramee (P. de)

And translated into English by WILLIAM KEMPE.

Imprinted at London by *Richard Field* for *Robert Dextar* dwelling in Paules Church yard at the signe of the brasen serpent. 1592.

TO THE RIGHT VVORSHIPFVLL SIR FRANCES DRAKE KNIGHT,

W. K. wisheth all good successe in this life, and eternall felicitie in the life to come.

FOrasmuch as the praises and commodities of Arithmeticke are not only learnedly set forth alreadie in diuerse treatises written of that art, but also by dayly experience well knowne to most men: it shall suffise for me in this place (right Worshipfull) to giue as it were but a glimse of the matter, briefly touching it only in generall. For who is there of what facultie or profession soeuer, that can either attaine to the exact knowledge of his art, or fitly and wittily exercise the same without the helpe of Arithmeticke? In diuinitie, how many questions are dissolued, and places plainely interpreted, especially by the benefit of this art? In ciuill policie, and in the seate of iudgement,

the golden rule of proportion is the law of equitie. In Phisicke, Hippocrates willeth his sonne Thessalus to learne Arithmeticke, that thereby he might iudge of the increasing, decreasing, continuing, and chaunging of diseases, as also compounding of medicines. In the workemanship of heauen and earth we are taught and see, that God hath made all things in number, in measure, in weight, that is to say, in a iust proportion. Astronomie doeth not onely measure the quantitie of celestiall creatures, but also numbreth their motions. What is Musicke in sownes, in harmonie, and in their spaces, concords, and diuerse sorts, but only Arithmeticke in hearing? Take away Arithmeticke, ye take away the merchants eye, whereby he seeth his direction in buying and selling; ye take away the goldsmiths discretion, whereby he mixeth his metalles in due quantities; ye take away the Captaines dexteritie, whereby he embattaileth his armie in conuenient order; finally yee take from all sortes of men, the facultie of executing their functions aright. Arithmeticke then teacheth vnto vs matters in diuinitie, iudgeth ciuil causes vprightly, cureth diseases, searcheth out the nature of things created, singeth sweetly, buyeth, selleth, maketh accompts, weyeth mettals and worketh them, skirmisheth with the enimie, goeth on warfare, and setteth her hand almost to euerie good worke, so profitable is

she

ſhe to mankinde. But though the vtilitie of Arithmeticke be ſo manifeſt, that no man can now doubt thereof: yet doubtleſſe many will demaund of me a reaſon, wherefore after that ſo many famous men haue ſufficiently declared all the myſteries of this art, I would nowe at length bring forth this litle pamphlet, as a candle to giue light at noone day. Whereunto I anſwere, that as diuerſe haue committed to writing very cunning and ingenious meditations vpon this art; ſo either for certaintie and truth of matter, or for perſpicuitie and plainneſſe of method, none haue done the ſame better, then hath Ramus, that worthie ornament of arts & all good learning in our time: whom as a moſt ſufficient & approued author I haue tranſlated and ſet forth in Engliſh, with ſuch generall and briefe precepts and familliar examples, as may be moſt conuenient and fit for the maiſter to teach and the ſcholler to learne, not only the art, but alſo the vſe of the art. And this I did at the firſt for the behoofe of ſome fewe perſons deſirous to learne this art; if others alſo may take profit therby, I would not withhold from them any thing that is good. But if any man be not fully content therewith, I ſhall requeſt him for the commoditie of his neighbours, to beare with that which otherwiſe he miſliketh. Now theſe my poore labours, though a homely preſent, I am bold to offer vnto your Worſhip, not as though

you had any neede of them: but partly to teſtifie my dutie and thankfulneſſe for your benefits beſtowed vpon me (ſeeing debt muſt be payd euen to kings and princes, haue they neuer ſo little need,) partly for that your Worſhip hauing by long exerciſe in waightie affaires, ſufficiently tried the veritie and valour of this art, can rightly iudge of theſe my doings, in noting that which is amiſſe, & allowing that which is otherwiſe. Wherfore hoping and yet neuertheleſſe deſiring, that of your accuſtomed curteſie you will fauorably accept this dutiful ſignificatiō of a welwilling mind, in moſt hartie maner I commend your Worſhip to the God of all knowledge and truth, who ſo lighten your heart, that ye may perfectly vnderſtand, & ſo ſtrengthen your ſpirit, that you may throughly accompliſh thoſe things which are for the aduauncement of his glorie, the preſeruation of our Prince and countrey, and the comfort of your owne conſcience. Amen.

Your Worſhips euer to
commaund in the Lord,

William Kempe.

LEt Iason, Tiphis, Hercules,
And all the men of fame,
Whom Greece is wont to bragge much of,
Loose now their former name:
For workes of greater price and praise
Our Drake hath tane in hand,
And eke perfourmd, ruling the ships
In flouds, and flouds in land.

W. Kempe.

THe irkesome drought that Plimmouth felt,
Full long allparts distrest,
Industrious Drake by bringing home
Fresh waters hath redrest.
What better thing effect might be?
What more of thankes and fame?
So great a worke did once aduaunce
Of Hercules the name.
Of Hercules the name did rise,
For killing Hydra fell,
Whose bodie through the lashing wounds,
In limbes and might did swell.
And shall the man not famous be,
That hath with valiant hand,
In like sort crusht the swelling neckes,
Of Spanish sturdie band?

Plus vltra certes had ere now
His loftie bonnet vayld,
Daunted with dent of thy sword Drake,
All courage in him quayld:
If carping lazie crue,
(Such are our times and dismall dayes)
Had not withstood thy braue attempts
And purpose good alwayes.

A. W.

THE

THE ART OF ARITHMETICKE.

CHAP. I.
The noting of numbers.

ARithmeticke is an art of numbering, whereof there are two partes, the one absolute, the other comparing. The absolute part considereth the simple nature of numbers. A number is that whereby any thing is numbered. And it consisteth either of an vnitie, which is the least number, or else of a multitude, whereof there cannot be so great but that there may be a greater. In numbers two things are to be considered, first the noting of them, and then the numbering. They are noted with these ten notes or figures: 1. 2. 3. 4. 5. 6. 7. 8. 9. 0. whose value is either single or increased: single when they stand alone, or else stand with others in the first place next toward the right hand, and then they signifie one, two, thrée, foure, fiue, sixe, seuen, eight, nine. As for 0 commonly called a cipher, it

hath no value at all, but serueth onely for increasing the value of other figures, wheresoeuer it stand. Now the value of figures is increased by thrée degrées, to wit, once, ten times, an hundred times: and these degrées are likewise repeated againe by course. As that which standeth in the first place towards the right hand hath his single value once, as I haue said: in the second place ten times, in the third place an hundred times. This is the first course. The seconde is of thousands. Wherefore in the fourth place yée shal number thousands once, in the fift place tē times, in the sixt place an hundreth times. Then followeth the third course, where yée shall note millions likewise once, ten times, an hundred times, and so increasing euerie place by ten infinitly. As for example one is thus noted 1, ten thus 10, an hundred thus 100, a thousand 1000. Againe twentie 20, thrée hundred 300, foure thousand 4000, fifty thousand 50000, sixe hundred thousand 600000. But if the summe be long written with manie figures, for the better understanding and vttering thereof, yée shall distinguish euerie course frō other by a prick thus 1 2 3 4 5 6 7 8 9 0. In the first course towards the left hand yée shall say that there is a thousand millions, in the next, two hundred thirtie foure millions, in the third, fiue hundred sixtie seuen thousand: in the last, eight hundred and nintie.

Here

Here you see that when numbers are to be valued, yee must begin at the right hand: but when their values are to be vttered, yee must begin at the left hand with the greatest numbers first. And thus these ten figures serue as an Alphabet, or a b c, to note and write anie number withall.

Chap. II.
Addition.

NUmbering is by two seueral numbers propounded to finde out the third, and vnlesse it may be finished all at once, it is done by induction of the parts: and then euery figure is considered as if he were alone: which when he serueth for the numbering next after him must be reserued in minde to auoide often blotting. Numbering is either single or manifold. Single numbering numbereth one number with another once, as Addition and Subduction. Addition is a single numbering, whereby one number is put to another, and so the totall is found: as put 1 to 1, the totall is 2, and adde 1 to 2, the totall is 3. adde 2 to 3, the totall is 5. adde 5 to 4, yee haue 9. and so must yee be exercised throughout all the alphabet of figures. Which numbering is easie because it is finished all at once: as also

is the addition of numbers continued in degrée one after the other. As if yée haue foure debters, of the which the first oweth 1000 l. the second 400, the third 50, the fourth 8. yée shall adde these numbers the one after the other, and say that the totall is 1458 l. But the addition of numbers discontinued, requireth a speciall meditation and forecast, which yée shall begin at the right hand, placing the numbers in such wise, that those be directly the one ouer the other, that are increased by like degrée, and draw a line vnderneath, noting vnder the same the totall that is gathered by addition. Whereof take this easie example first. A debter oweth to one man 365 l, to another 214, he would know the whole summe of both. First set downe the numbers in due order thus,

$$\begin{array}{r} 365 \\ \underline{214} \end{array}$$

Then beginne with 4 and 5, which are 9. write therefore 9 vnderneath: in the second place 6 and 1 added make 7, write that vnderneath also. Lastly note 5 vnderneath comming of the addition of 3 and 2, and the induction will be thus,

$$\begin{array}{r} 365 \\ \underline{214} \\ 579 \end{array}$$

In this example the third number is found out of the two that are propounded, by an induction of the parts, which are considered euerie one as if they were alone. Now another example wherein the reseruing of a number in minde must be vsed. As let vs adde 56789 to 1234. The numbers being set

downe

downe after this fashion,

I will begin thus, 4 and 9 are 13 Here I haue two figures to be noted in two places, the first toward the right hand vnder the figure of that place, and the other to be reserued for the place following. Wherefore I note 3 vnderneath, reseruing 10 for one of the place following, and in the place following I say 1 and 3 are foure, 4 and 8 are 12, I note 2 vnderneath and reserue 10 for 1 of the place following, as before: then 1 and 2, and 7 are 10 for 1 of the place following: wherefore I note 0 vnderneath, keeping 1 in minde, the which 1 and 1 and 6 in the next place being 8 by addition, I write 8 vnderneath: then finding 5 alone, I write the same alone likewise. So haue I ẏ whole sum standing thus by induction.

56789	56789
1234	1234
	58023

There may be an addition of moe numbers then two, notwithstanding two onely are considered seuerally, which being first added togither must then as one number be added to the third. As if it be demanded how long it is since the world began: and answere be made out of the Bible, that God created heauen & earth 1656 yeares before Noahs floode, that the floode was 793 yeares before the giuing of the law, that the law was giuen 1525 yeares before Christ came in the flesh, and that this is the 1591 sithence

the birth of Christ. Adde these foure numbers togither, the summe of induction, shewing that the worlde was created 5565 yeares agoe, will be in this manner.

The like must be done how many numbers soeuer be propounded. And so much of Addition.

1656
793
1525
1591
5565

Chap. III.
Subduction.

Subductiō is a single numbering, wherby one nūber is taken from another, & so the residue or rest is foūd. As take 1 from 2, the rest is 1, take 2 from 6, the rest is 4. take 4 from 9, there remaineth 5. For here must be the same exercise of subduction in single figures that was before of addition. Now let vs take an example wherein the whole number cannot be taken from the whole at once. As if a man that was 579 l. in det, haue paide thereof 214 l., how much doth there remaine to pay? The numbers being orderly set downe, the lesse number that yée take away vnder the greater from whence yée take it, begin at the left hand cōtrarie to the order of addition, noting the residue

stoue aboue, & cancelling or defacing 579
the former numbers thus, 214

First take 2 from 5, there remaineth 3, which write ouer the same place, and cancell 5 and 2 with a dash: then take one out of 7, there is left 6, note 6 ouer, cancell 7 and 1. Finally, take 4 from 9, yee leaue 5, write 5 ouer, deface 9 and 4. So yee finde 365
the rest of the debt to be 365, the 579
whole induction will stand thus, 214

Here the third number is found out of the two that are propounded, wrought by an induction of the partes in such sort as if they stode alone. Now another example of a number reserued in minde, as when the figure next following which must be taken away, is greater then the figure ouer it: then of the rest going before, must one be kept in minde which shall increase the figure following with 10. As in that example before: if I take 1656 from 5565 to know how long sithence was Noahs flood: here when I haue taken 1 from 5, I may not write 4 aboue because 6 which followeth is greater then 5 the figure ouer him: but I write 3 aboue, reseruing 1 in minde for 10 of the next place, and then I take 6 from 15, noting the residue 9 aboue, because the figure following is lesse then that ouer it: further taking 5 from 6, there resteth 1, which I may not write, for that 6 following is greater then 5 ouer it,

wherefore I write 0 aboue, and reserue 1 for 10 in the next place. Last of all I take 6 from 15 and note the rest 9 aboue. And so finde it to be 3909 yeares agone since the floode, the the whole induction standing thus,

3909
5565
1656

This is the right way of Subduction, as shall appeare afterward in greater numbering: neither may the former figure be taken away before it be considered from whence that which followeth may be taken. Now if the number which must be subducted, or the number from whence the subduction is made consist of manie termes: First these termes shalbe brought into one summe by addition. As if a marchant haue bought 20605 bushels of wheat in one bargaine, and 30403 in another, and hath promised 42026 bushels thereof to another man, how much hath he kept for himselfe? Adde 20605 and 30403, the totall is 51008, take thence 42026, there will remaine for the merchant 8982. This example hath greater difference then the former, and will stand in this forme:

30403	8982
20605	55008
51008	42026

And here it shall not be amisse to exercise the learner in such questions as are answered mutually by Addition and Subduction. As from what number may 9 be taken that there may remaine 6? Adde 9 to 6, yee haue

15 the

15 the number that was demaunded. To what number muſt 8 be added that the whole may be 13? Take 8 from 13, the reſt 5 ſheweth the number. And hitherto of ſingle numbering.

CHAP. IIII.
Multiplication.

Manifold numbering numbereth one number with another ſo often as the number is propounded: and it is either Multiplication or Diuiſion. Multiplication addeth the multiplicand ſo often as the multiplier containeth an vnitie, and ſo the ofcome is found. Though an vnitie doe augment a ſumme by adding, yet by multiplying it augmenteth nothing. Howbeit all other numbers do augment by multiplying, and ſome of them ſo much that they ſurmount the ſum of their Addition by farre. Wherefore the exerciſe of multiplying the ſingle figures one with another muſt be applyed the more diligently. As twiſe two are 4, thriſe three are 9, foure times foure 16, fiue times fiue 25. Againe twiſe three 6, twiſe foure 8, and ſo throughout the whole Alphabet of figures. But if anie of the greater figures ſeeme hard to young beginners, they may practiſe in minde to multiplie by the partes

and then adde the ofcomes togither. As for foure times nine, firſt take twiſe 9 which is 18, then twiſe 9 more maketh another 18, both ofcomes added togithers make 36 the whole ſumme. Yea further a man may multiplie by the partes of both numbers, and ſo adde the particular ſummes togither, as 8 by 9 thus

$$\begin{array}{r} 45 \\ \underline{44} \\ 20 \\ 16 \\ 20 \\ \underline{16} \\ 72 \end{array}$$

But now an example of multiplying ſome greater number. As how many pound will ſerue the yearely wages of 567 ſouldiers, when as euerie ſouldier hath 7 l. by the yeere? Here the multiplicand being the greater number is more fitly placed ouer, the multiplier vnder next towards the right hand, for of that ſide muſt yee begin as in Addition, drawing a line vnderneath, and then muſt the three vpper figures be multiplied ſeuerally by the multiplier, and ſo the ofcome noted vnder the line thus.

$$\begin{array}{r} 567 \\ \underline{7} \end{array}$$

Seuen times 7 are 49, note 9 vnderneath, and keepe 40 in minde for 4 of the place following. Seuen times 6 are 42, and foure reſerued are 46, note 6 keeping 40 againe for foure in the next place. Seuen times 5 are 35 and 4 reſerued are 39, which ſet downe the one after the other, ſeeing there remaine no more figures to be multiplied, The ſumme of the induction will be thus.

$$\begin{array}{r} 567 \\ \underline{7} \\ 3969 \end{array}$$

Here

Here we see the finding out of the third number, the inductiō of the parts, the figures considered alone, & the reseruation of numbers in memorie: The like shalbe done by the partes as well of the multiplier, as of the multiplicand, whereof take this larger example, 3060 is to be multiplied by 204, the induction of the partes will make 624240 thus,

3060
204
12240
61200
624240

Where we see that though the cypher multiplied, either by a cipher, or by any other figure make nothing, yet must he be noted in the beginning to increase the figures following: but standing in the mids of the multiplier, there is no neede to repeat him by multiplication. Also if numbers ende with ciphers, we may vse a briefe way in writing the ciphers but once, as 8600 must be multiplied by 350 this way.

8600
350
430000
25800
3010000

If any of the termes haue 1 in the first place the rest being ciphers, adde the ciphers to the other terme, and the multiplication is finished. As 100 must be multiplied by 37, adde 00 to 37, yee haue 3700. These are sufficient for the multiplying of any number, be it neuer so great.

Chap. v.
Diuision.

Diuision is a manifold numbering, whereby the diuider is taken from the diuidend so often as he is contained therein, and so the quotient is found. As an vnitie doth not increase a number by multiplying, no more doth it diminish a number by diuiding, though it diminish by subducting. For if yee diuide 6 by 1, the quotient will be 6 stil, equall with the diuidend: but if yee diuide 6 by 6, the quotient will be 1, that is the first and smallest part of the diuidend: if by 2, the quotient will be the second part of the diuidend: if by 3, the quotient wil be the third part: if by 4, the fourth part, and so forth. So that as in Addition the totall is found out, in Subduction the residue, in multiplication the ofcome: euen so in diuision there is found out the quotient namelike to the diuider. Wherefore Diuision is answerable to Multiplication, but according to a backward comparison. For in multiplication as an vnitie is to the multiplier, so is ye multiplicand to the ofcome: contrariwise in diuision as ye diuidend is to the diuider, so is the quotient to an vnitie. The exercise of learning, which we haue seene in the former sorts of

of numbering, must here most of all be vsed and applyed in learning perfectly throughout the Alphabet of figures, what number and by what quotient euerie one of them doth diuide, which is knowne by the comparison of Multiplication. For by what number a multiplier maketh an ofcome, by the same shall he diuide the said ofcome. As an vnitie maketh any number in multiplying by the same number, therefore euerie number diuideth himselfe by an vnitie, and an vnitie diuideth euerie number: thrise 4 maketh 12, therefore 3 diuideth 12 by 4, and 4 diuideth 12 by 3. nine times 7 maketh 63, therefore 9 diuideth 63 by 7, and 7 diuideth the same nũber by 9, and so in all numbers either single or increased. Now let vs take an example of a greater diuision, wherein the diuidend must be placed ouer, the diuider vnder in the place next towards the left hand, as in Subduction noting the quotient at the right side by it selfe behinde some line, as 8766 li. must be diuided equally to 6 persons: therefore I

<table>
<tr><td>place the diuidend 8766 ouer, the diuider 6 vnder at the left side in this wise.</td><td>8766
6 (</td></tr>
</table>

Out of 8 I cã take 6 but once, there remaining 2, wherefore I note 1 for the quotient,

<table>
<tr><td>and hauing crossed out 8 and 6 I write the residue 2 aboue, so will be the first induction.</td><td>2
8766
6 (1</td></tr>
</table>

Secondly I set forth the diuider into the next place, where because I may take 6 out of 27 foure times 3 remaining, I note 4 for the quotient, then for fearing of forgetfulnes and other failing, I multiplie 4 the quotient by 6 the diuider, which make 24, which being takē from 27 there resteth 3, I write the rest aboue cancelling both 6 and 27, so is the second induction.

2 3
8766
66 (14

Thirdly I set forth the diuider 6 into the next place of 36, whence I may take 6 sixe times nothing remaining, I note 6 in the quotient, and for more certaintie multiplie 6 by 6, and seeing the ofcome is equall with that ouer, I cancell both 36 and 6 the diuider, noting no remnant at all: so haue I the third induction.

2 3
8766
666 (146

Lastly I bring forth the diuider 6 into the place of 6, from whence I may take 6 once, there resting nothing: therefore noting 1 in the quotient I cancell the diuidend, and the diuider, and so haue the whole induction standing thus.

2 3
8766
6666 (1461

So I finde that 8766 being diuided into 6 equall partes, the quotient will be 1461, which is the iust portion that ech of these 6 persons must haue, and therefore the number that was sought for in this diuision. But if the

the diuider consist of moe figures then one, they shalbe considered al togither as one, and so the whole diuider by his partes equally must be taken from the vpper figures of the diuidend, as often as he is contained therein. Whereby is plainly sene, that the right way of subduction beginneth at the left hand, as hath bin said: for an example whereof I will diuide 192 pence by 12 d. which is a shilling, that the quotient, which is alwayes name-like to the diuider, may shew the shillings contained in the diuidend: the numbers being orderly set downe in this wise,

1 9 2	
1 2	(

First 1 may be once taken from 1, and so often may 2 from 9, and there will rest 7, noting therefore 1 for the quotient, & hauing defaced 19 and 12 I write the rest 7 above: so is the first induction.

	7	
~~1~~	~~9~~	2
~~1~~	~~2~~	(1

Secondly I bring forth the diuider into the next place of 72, where I see that I may take 1 seuen times from 7, but I cannot take 2 so often from the rest being but 2, wherefore that the parts of the diuider may be equally subducted, I take 1 sixe times from 7, & from the rest being 12 so often doe take 2, I write therefore 6 for the quotient, multiplie thereby the diuider, and take the ofcome being 72 from 72, the diuidend, there remaineth nothing: wherefore I cancell both the one and

the other: ſo is the whole indu-
duction,

1
7
1 9 2
1 2 2 (16
1

In the firſt induction of this example ỹ ſecond figure might haue bene taken away more often then the firſt figure: in the ſecond induction, the firſt figure might haue bene taken away more often then the ſecond: but there muſt be alwayes obſerued herein an equalitie of Subduction. Another example which ſhall onely be ſet forth with the figures: as, let 841 ł be ſhared amongſt 29 ſouldiers.

Firſt induction	Whole induction
2	2 8
4 6	4 6
8 4 1	8 4 1
2 9 (2	2 9 9 (29
	2

In the ſecond induction of this example 2 might haue bene taken thirtéene times out of 26 the vpper number: but no figure of the diuider may be taken more often then nine times, becauſe a greater number then 9 cannot be expreſſed with one note. If the diuider be greater then ỹ firſt place of the diuidēd, let it be moued forth into the ſecond place. As if 414 muſt be diuided by 46, I cannot take 46 from 41, I put forth therfore the diuider into the ſecond place, and ſo finde the quotient to be 9 thus,

5
4 1 4
4 6 (9

If in anie place after the first, the diuider happen to be greater then the diuidend, then a cipher must be noted in the quotient, and so procéed, as if 7344 be propounded to be diuided by 36, the quotient will be 204. If so be a voide place happen in the figure left in the midds, a cipher must be put there, as commeth to passe in diuiding 648 by 54 thus,

1 0
648
54 (1

If there be ciphers in the end of the diuider, it is a briefe way to set them downe at the first vnder the end of the diuidend, working onely with the figures of value vntil yée come to the ciphers. As if yée diuide the 5000 men by the 200 peny worth of bread, which Philip said would not suffice for euerie man a little, to wit, one penyworth for 25 men as appeareth.

First induction,	Whole induction,
1	1
5000	5000
2 00 (2	2200 (25

Finally, if the first figure of the diuider be 1, the rest ciphers, take from the later end of the diuidend as manie figures as there be ciphers in the diuider, and the diuision is finished, as 6400 diuided by 100 the quotient is 64. This art of Diuision thus declared is sufficient to diuide anie number be it neuer so great, as if 974074065210 be diuided by 789, the quotient will be

1234567890. Thus we see the variety of so many sorts of numbering in Division to require a carefull minde, a sure memorie, and especially a readie and a true hand: and therfore to make the learner more diligent, it is good sometime to delight his fantasie with those questions that haue in them all sorts of numbering: as admit that thou didst fall into the hands of thrée théeues, the one after the other: the first tooke from thee thrée quarters of thy money, and gaue thee 6 shillings againe: the second tooke away a quarter of that was left and 3 s̄, the third tooke halfe of that was left, and gaue 1 s̄ againe; when thou wast escaped them all thou hadst 7 s̄: how much hadst thou in the beginning, and how much did euerie one take from thee? From 7 take 1 that the third restored thee, there remaineth 6, this is halfe: the third tooke away the other halfe, therefore double it yée haue 12, hereunto adde the 3 that the second had aboue a quarter, it is 15. this is thrée quarters, therefore the quarter that the second had is 5, adde both togither it maketh 20, from whence take the 6 that the first gaue againe, there resteth 14, this is one quarter, the first had the thrée other quarters, so that the whole was foure times so much as this. Multiplie it therefore by 4, it maketh 56, this summe you had in the beginning, hereof giue thrée quarters 42 to the first, and

let

let him restore againe 6, he carieth away 36, 20 remaining, whereof giue a quarter 5 to the second, and also 3, he carrieth away 8, 12 remaining: thereof giue halfe 6 to the third and let him restore 1, he carrieth away 5 and leaueth 7.

In this question you see Addition, Subduction, Multiplication and Diuision: but in that following is onely Multiplication and Diuision. As in how many dayes and for what wages will 125 men digge and cast a trench of 27 miles long, when as one man worketh 3 pases in a day for 6 d? First by multiplication we see, that (because a mile is 1000 pases) the 27 miles are 27000 pases, and that the 125 men worke 375 pases in a day: then by diuiding the 27000 pases of the whole trench, by 375 the pases wrought in a day, we vnderstand that the quotient 72 sheweth the number of the dayes wherein the 125 men wil ende the worke. Now seeing three pases are wrought for 6 pence, 120 are wrought for 20 s, that is one pound, diuide then 27000 by 120, the quotient 225 sheweth the pounds of the wages.

CHAP. VI.

Numbers euen and odde.

NOw by Diuision a number standeth whole or else is broken into parts. Againe whole numbers

haue two differences arising likewise of Diuision. The first difference is wherby a number is euen or odde. An odde number is that which is vndiuidable by two, as 1.3.5.7.9. An euen number is that which is diuidable by two, as 2.4.6.8. An euen number is euenly euen, or oddely euen: an euenly euen number is that which is diuidable by an euen number into an euen, as 4 is diuided by 2 into 2, 8 diuided by 2 into 4. Of this sort are all numbers doubled of two, as 4.8.16.32.64. which number is commodious for the vse of warres to change the rankes in an armie, as are 32768, or 16384. An oddely euen nūber is that which is diuidable by an odde number into an euen. So 6 is diuided by 3 into 2, 12 diuided by 3 into 4, 30 by 5 into 6: and this is the first difference that commeth by Diuision.

Chap. VII.

Numbers vncompound and compound.

The second difference is of one number alone or of mo numbers togither. The difference of one number alone is of an vncompound number, and of a compound. An vncompound number is vndiuidable by another number of multitude, as in the Alphabet of figures, 1.2.3.5.7. one is

is diuided only by it selfe, the rest may be diuided euerie one by himselfe, and also by 1: but by any other number of multitude they are vndiuidable. A compound number is diuidable by another number of multitude: as 4 may be diuided by 2 into 2, 6 diuided by 2 into 3. 8 diuided by 2 into 4. 9 by 3 into 3. Of this sort some are compound but one way, & therefore but one way diuidable, as 4 onely by 2, 9 onely by 3: some mo wayes, and therfore mo wayes diuidable, as 6 by 2 and 3, 8 by 2 and 4. And this number that is manie wayes diuidable hath oftentimes notable vse, when such numbers are sought for, as will admit verie manie iust diuisions. So haue the Astronomers chosen the number of 60 to measure the space of a degrée, and likewise of an houre, whose diuiders are 1. 2. 3. 4. 5. 6. 10. 12. 15, 30. 60. For the same cause also our ancesters did chose the same number for the pence of their crowne: and the same number doubled, to wit, 120 for their hundred in diuers things: againe the same number foure fould, to wit 240 for the pence of their pounde: yea for this cause haue they chosen not onely 12 for the pence of their shillings, but also by it to reckon many thinges as by dosen, and by grosse a dosen dosen. Wherefore to know how often a number may be diuided this rule is vsed. The vncompound diuiders

of any number propounded, are the next after an vnitie that diuide as often as they can the number propounded, and the quotient thereof, and euerie quotient of the quotient. And the compound diuiders are the ofcomes of the last of the vncompound by the last saue one, and of that which followeth by both, and by the ofcome of both, and so likewise of the rest by all the former. As for example, I take 120 the English hundred. First 1 doth not diminish this number, 2 a number vncompound diuideth it, and the quotient is 60, which being diuided by 2 againe, the quotient is 30, which also being diuided by 2, the quotient is 15, and 15 being diuided by 3 an vncompound diuider likewise the quotient is 5. Here the vncompound diuiders are 1. 2. 2. 2. 3. 5. which being multiplied togither make vp the number propounded. Now come to the compound diuiders, and first multiplie the last by the last saue one, to wit, 5 by 3, the quotient is 15. This is the first order, 3. 5. 15. Secondly multiplie these thrée numbers by 2, yée make the second order thus, 2. 6. 10. 30. Thirdly these former orders multiplied by 2 make the third order 4. 12. 20. 60. Lastly multiplie by the other 2, yée haue the fourth order 8. 24. 40. 120. Therefore the diuiders of 120 are 1. 2. 3. 4. 5. 6. 8. 10. 12. 15. 20. 24. 30. 40. 60. 120. and these are seuen diuiders more then 100 can admit, for in 100 yée cannot find

neither

neither third part, nor ſixt part, nor halfe quarter, nor twelft part, nor diuers other parts that are found in 120. wherefore this number is more fit a great deale for parting and ſharing, as falleth out in ſuch things as are reckoned by 120 for the hundred, and ſo is the ſecond difference of one number alone.

CHAP. VIII.
Numbers vncompound betweene themſelues.

THe ſecond difference of manie numbers togither ariſeth of the former: for it is of numbers that are vncompound betwéen themſelues, or elſe compound betwéene themſelues: whereof there is notable vſe as ſhall appeare hereafter. Uncompound betwéene themſelues are ſuch numbers as cannot euerie one be diuided by one and the ſame number of multitude, as are 2 and 3, 4 and 7. 6, 8 and 9. Theſe are knowne by ſubduction and diuiſion: for if of two vnequall numbers, the leſſe being continually taken from the greater, do leaue no number of multitude which may diuide his former number, they are vncompound betwéene thẽſelues. So 4 and 7 are vncompound betwéene themſelues, for take 4 from 7 there remaineth 3, which diuideth not 4 the former num-

ber, then take 3 from 4, there remaineth 1 which diuideth the former number 3, but it is no number of multitude, for an example 32 and 9, 34 and 25 shalbe tried by a continuall subduction to be vncompound by themselues thus,

32	34
9	25
5	9
4	7
1	2
	1

If an vncompound number diuide not the number propounded they are vncompound betweene themselues, as 2 an vncompound number diuideth not 9, therefore 2 and 9 are vncompound by themselues. Now as by subduction and diuision vncompound numbers betweene themselues are knowen, so are they made by Addition and Multiplication. For if two numbers be vncompound betweene themselues, the totall of them is vncompound to either of them: and contrariwise, as 15 the totall of 7 and 8 is vncompound to 7 and 8. And contrariwise seeing 15 is vncompound to 7 and 8, 7 and 8 are vncompound betweene themselues: this for addition. Further if two numbers be vncompound to the third, the ofcome of both shalbe vncompound to the same. As 3 and 4 are vncompound to 5, and 12 the ofcome of them is vncompound to 5. Hereof follow two rules. The first is if two numbers be vncompound betweene themselues, the ofcome of the one by it selfe shall be vncompound to the other, as in 5 and 4, 25 the ofcome of 5 multiplied by it selfe is vncompound

compound to 4. The second rule is, that if two couple of numbers be uncompound betwæne themselues, their ofcomes shalbe uncompound betwæne themselues, as in 7 and 5. 4 and 3. to wit 7 is uncompound to 4 and 3. and also 5 to 4 and 3. therefore 35 and 12 the ofcomes of ech couple are uncompound betwæne themselues. Of these two rules followeth this conclusion, that if two numbers be uncompound betwæne themselues, the ofcomes of ech of them by himselfe, and againe by the ofcomes shalbe still uncompound betwæne themselues, as

3.	9.	27.	81.	243.
4.	16.	64.	256.	1024.

This inuention bringeth singular commoditie in progression, as shall be séene in his place.

Chap. ix.

Numbers compound betweene themselues and their greatest common Diuider.

Numbers compound betwéen themselues are such as are diuidable by one and the same number of multitude. As 4 and 6 are compound betwæne thēselues because they are both diuidable by 2 a number of mul-

titude. So 3 and 9, because they are both diuidable by 3, and likewise 5 and 5. So that of numbers compound betwæne themselues sometimes both are compound, somtimes but one, and sometimes neither of them. Here two things are to be considered, the greatest common diuider, and the least common diuidable. The greatest common diuider is the first remainer in a continuall subduction that diuideth the former number. As in 6 and 14, the greatest common diuider is 2, because by a double subduction, he is the first remainer that diuideth 6 the former number. Hereof it followeth that euery number which diuideth another is the greatest common diuider of them both. As 3 diuideth himselfe and diuideth 9, therefore he is ths greatest common diuider of 3 and 9. the like commeth to passe in a greater example, as

219
126
93
33
27
6
3

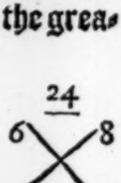
By the same way may be found out the greatest common diuider of compound numbers, how many soeuer they be. For the diuider of the two former being found shall serue as the diuidable for them; of the which and of the next number yée shall séeke the common diuider, as in 12.18.27. the greatest common diuider is 3. for first 6 is the greatest diuider of 12 and 18, and then 3 is the greatest diuider of 6 and 27. Now the greatest diuider sheweth in the quotient the least

diuider, as 18 being diuided by 9, yéeldeth 2 in the quotient. Againe the greatest diuider of numbers compound betwéene themselues yéeldeth in their quotients numbers vncompound betwéene themselues, as 18 and 24, whose greatest common diuider 6 hath in the quotients 3 and 4, numbers vncompound betwéene themselues.

Chap. X.

The least common Diuidable.

The least common diuidable of two numbers is the ofcome of the one by the others diuider namelike to the greatest common diuider, as the least common diuidable of 6 and 8 is 24, for the greatest common diuider of 6 & 8 are 2, and therefore their namelike diuiders are halfe of either of them, to wit 3 and 4, but the ofcome of 8 by 3, or of 6 by 4 is 24 ẏ least common diuidable of 6 and 8. The example is thus,



Hereof follow two consequents, the first that the ofcome of two vncompound numbers betwéene themselues is the least diuidable of both; as 15 the ofcome of 5 and 3 is the

least diuidable of 5 and 3, whereof behold the example,

The second consequent is that the diuidable of any one is the least diuidable of both, as 6 is diuidable of 3, therefore he is the least diuidable of 6 and 3, as the example sheweth,

By the same way is found out the least diuidable of how manie soeuer. For the least diuidable of two being found, must be conferred with the rest to find the least diuidable of it selfe and of the rest. As the least diuidable of 3.4.8 is 24. for the least diuidable of 3 and 4 is 12, and the least diuidable of 12 and 8 is 24. Hereof followeth that the least diuidable of the names of parts, is that which hath the parts: as the least diuidable that hath one second, one third, one fourth is 12, being the least diuidable of 2 3 and 4, and the least that may be diuided into two parts, thrée parts, foure parts.

Chap. XI.

The noting of parts and particles.

HItherto of whole numbers with their diuers sorts arising of diuision: now follow the parts and péeces of a number, to wit of an vnitie, which hap-

pen

pen to be, when the terme of the diuider is greater then the diuidend. Theſe parts require a peculiar kind of noting and alſo of reduction before their numbering. For they are noted with two termes, hauing a line betwéene them: the vpper is called the numberer, the vnder is called the namer. So if 11 s̄ be diuided to 4 perſons, the quotient will be 2 and thrée quarters which is thus noted $\frac{3}{4}$, ſhewing that beſides the two ſhillings there is thrée quarters of a ſhilling, to wit, 9 pence for ech mans porcion: for here 4 nameth the parts of a ſhilling, and 3 numbereth them. So if 4 s̄ ſhould be diuided to 6 men, the quotient will be $\frac{4}{6}$, that is foure times the ſixt part of a ſhilling, to wit 8 d. Again 9 s̄ diuided to 4 perſõs thus,

1
$\cancel{9}$
4 (2 $\frac{1}{4}$

That is 2 s̄ and 3 d, in which caſe the whole number in the quotient ſheweth the parts of the diuidend: but the péeces ſhew the partes not of a multitude, but of an vnitie: and as much as the numberer is leſſe then the namer, ſo much is there wanting of an vnitie. As $\frac{1}{4}$ lacketh $\frac{3}{4}$ of an vnitie, & therfore if they be both equall, the value is iuſt one, as $\frac{4}{4}$ is 1. As for particles and the parts of parts, onely the leaſt of them haue a line betwéene them and not the reſt: for thrée quarters of two third parts of one halfe are thus noted. $\frac{3}{4}\,\frac{2}{3}\,\frac{1}{2}$. As if a man leaue his land equally to his two daughters, and one of them haue thrée daugh-

ters for her heires, of whom the firſt buyeth the ſeconds part, and the ſame leaueth foure daughters for her heires, and the firſt of them buyeth the parts of the ſecond and the third, this niece is to haue of her great grãdfathers inheritance $\frac{2}{4}\,\frac{2}{3}\,\frac{1}{2}$.

Chap. XII.
Reduction before ſingle numbering.

Thus haue yée the noting of the partes of an vnitie, their reduction ſtandeth either without numbering or elſe muſt neceſſarily be ioyned with numbering. The firſt ſerueth onely for ſingle numbering, and it is the reduction of the termes alone, or elſe of whole numbers and of parts. The termes being compound betwéene themſelues are reduced to their leaſt proportionall termes by diuiding them by their greateſt common diuider. So $\frac{12}{16}$ diuided by 4 their greateſt common diuider come to $\frac{3}{4}$. This reduction bringeth great eaſineſſe in handling of parts. Wherein to number by termes compound betwéene themſelues, is as great abſurditie as to refuſe the neareſt way and go farre about. Next is the reduction of whole numbers & of parts. The reduction of whole numbers to partes is to multiplie them

them by the namer of the partes. So the reduction of foure ſhillings to pence is to multiplie 4 by 12 (becauſe a penie is $\frac{1}{12}$ of a ſhilling) and they will be 48 pence. The reduction of partes is either to whole numbers, or to partes. The reduction of partes to whole numbers is to diuide the parts by their namer: as to reduce $\frac{48}{12}$ to whole numbers, is to diuide 48 by 12, and the quotient 4 ſheweth the whole 4 ſhillings. The reduction of partes to partes is to a couple of proportionall parts, or elſe to one equall. The reduction of partes to a couple of proportionall partes is to take the leaſt diuidable of the namers for the common namer, and to multiplie interchangeablie the numberers by their namelike parts. So $\frac{3}{4}$ and $\frac{5}{6}$ are reduced to $\frac{9}{12}$ and $\frac{10}{12}$ thus,

$$\begin{array}{cc} 9 & 10 \\ \frac{3}{4} & \frac{5}{6} \\ 2 & 3 \\ \multicolumn{2}{c}{12} \end{array}$$

If there be manie ſeuerall partes, firſt two are to be reduced, and then the ſame ſo reduced and added, muſt bée conferred in like ſort with the next partes. As in $\frac{1}{2}\,\frac{2}{3}\,\frac{3}{4}$. Firſt $\frac{1}{2}\,\frac{2}{3}$ reduced and added are $\frac{7}{6}$, the which reduced with $\frac{3}{4}$ make $\frac{9}{12}\ \frac{14}{12}$. This reduction ſheweth of two ſeuerall vnequall partes, which is the greater. As $\frac{3}{4}$ is greater then $\frac{5}{7}$, becauſe by reduction $\frac{3}{4}$ will be $\frac{21}{28}$ and $\frac{5}{7}$ but $\frac{20}{28}$. The reduction of partes to one equall part, is the reduction

of particles and the parts of parts, which is done by multiplying the numberers betwéen themselues, and the namers betwéene themselues, as $\frac{3}{4}\frac{2}{3}\frac{1}{2}$ reduced come to $\frac{6}{24}$, that is $\frac{1}{4}$ Hereby is found out the value of particles, yea the particles of whole numbers also, as $\frac{5}{6}$ of 30 l. is 25 l. for $\frac{5}{6}$ and $\frac{30}{1}$ being multiplied make $\frac{150}{6}$, which by reduction is 25. Reduction which must necessarily be ioyned with numbering, serueth for manifold numbering, as shalbe obserued in his place.

Chap. XIII.
The numbering of parts.

NOw after the parts noted and thus reduced, followeth the numbering of them, which is easie if it be onely of parts, as in Addition and Subduction the namers being like, onely the numberers are dealt withal, as $\frac{2}{7}\frac{3}{7}$ are $\frac{5}{7}$, and $\frac{2}{7}$ taken from $\frac{3}{7}$, there remaineth $\frac{1}{7}$. Concerning manifold numbering, first multiplication multiplieth ÿ namers ech with others, & so likewise the numberers, as $\frac{2}{3}$ and $\frac{2}{5}$ multiplied are $\frac{4}{15}$. The reduction here ioyned with multiplication is that if any numberers and namers be compound betwéene themselues, they must be interchangeably reduced to their vncompounds, yea altogither omitted

if

if they be equall, as in $\frac{2}{3}\,\frac{7}{8}$, reduce 2 and 8 to 1 and 4, & multiplie $\frac{1}{3}\,\frac{7}{4}$, which make $\frac{7}{12}$, for it is all one to multiplie $\frac{2}{3}$ by $\frac{7}{8}$ and $\frac{2}{8}$ by $\frac{7}{3}$. So omitting the equals, as $\frac{3}{4}\,\frac{4}{5}$, 4 and 4 omitted $\frac{3}{5}$ is the ofcome. So in a longer example $\frac{2}{3}\,\frac{3}{4}\,\frac{4}{5}\,\frac{5}{6}$, the equals omitted yee haue $\frac{2}{6}$ which reduced againe is $\frac{1}{3}$. Diuision of parts by parts is to multiplie interchangeablie the numberers by the namers, placing the ofcome according to the termes of the diuidend, as $\frac{3}{4}$ diuided by $\frac{2}{3}$ are $\frac{9}{8}$, $\frac{2}{3}$ diuided by $\frac{3}{4}$ are $\frac{8}{9}$. Here the reduction is of the numberers compound betweene themselues, and likewise of the namers betweene themselues: as if $\frac{3}{4}$ must be diuided by $\frac{5}{6}$, for 4 and 6 take 2 and 3, and so by diuision make $\frac{9}{10}$. Also if $\frac{3}{4}$ must be diuided by $\frac{6}{7}$, take 1 & 2 for 3 and 6, and the quotient wil be $\frac{7}{8}$. Againe diuiding $\frac{4}{5}$ by $\frac{8}{15}$, for 4 and 8 take 1 and 2, for 5 and 15 take 1 and 3, the quotient will be $\frac{3}{2}$. In this manifold numbering of parts, the ofcome wareth lesse by multiplication, and the quotient wareth greater by diuision: both which are according to the analogie of multiplication and diuision: for here an vnitie is greater then ye multiplier, therefore the multiplicand is greater then the ofcome. So contrariwise an vnitie is greater then the diuider, therefore the quotient is greater then the diuidend. This is the numbering of partes alone: now followeth the numbering of partes with whole numbers.

Addition changeth nothing : as adde 3 to $\frac{1}{4}$, it is 3$\frac{1}{4}$. So is the addition of whole numbers with partes to whole numbers with parts: as if 6 l, 19 s, 148 d must be added to 7 l 59 s, 265 d. First collect the pence 413 that is $\frac{413}{12}$ shillings by diuision found to be 34 s and 5 d, wherefore note 5, and keepe 34 for the next: then 34 s, 19 s and 59 being added, the totall is 112, that is $\frac{112}{20}$ poundes which by diuision is 5 l, 12 s: note 12 and reserue 5, which being added to 6 and 7, yee haue 18 pounds and the summe of this addition will be in this wise,

6 l	19 s	148 d
7	59	265
18	12	5

As for subduction, first an vnitie or moe of whole numbers being reduced to parts, take the numberers from the numberers. As if $\frac{2}{5}$ must be subducted frō 3, take 1 from 3 for $\frac{5}{5}$, frō whence take $\frac{2}{5}$ there will remaine in all 2$\frac{3}{5}$. So if 3$\frac{2}{5}$ must be taken from $\frac{21}{5}$, there will remaine $\frac{4}{5}$. So is the subduction of whole numbers and partes mixt togither. The subduction of whole numbers and parts vnmixt may be done after the same way that addition is: but if the parts exceed an vnitie they shall be reduced to their vnities by diuision. As if 6 l 49 s, 167 d must be subducted from 18 l 12 s 5 d, first reduce 167 d to 13 s and 11 d, then 13 s added to 49 s the totall is 62 l, which reduced is 3 l 2 s, now adde 3 to 6 l, the totall is 9 l: so then take 9 l 2 s 11 d from 18 l

12 s 5 d, the reſt is 9 l, 9 s 6 d the ſumme of this ſubduction ſtandeth thus,

9 l	9 s	6 d
18	12	5
9	2	11

The multiplication of whole numbers by parts putteth 1 vnder for the namer, and multiplieth as before. So $\frac{4}{1}$ multiplied by $\frac{2}{3}$ make $\frac{8}{3}$. But a whole number with partes may be multiplied either by the whole number alone, or elſe with the partes, and that both ſeuerally and ioyntly. So $5\frac{3}{7}$ is multiplied by $4\frac{2}{3}$. The firſt multiplication of 5 by 4 maketh 20, the ſecond of 5 by $\frac{2}{3}$ maketh $\frac{10}{3}$, that is $3\frac{1}{3}$. The third of $\frac{3}{7}$ by 4 maketh $\frac{12}{7}$, that is $1\frac{5}{7}$. The fourth of $\frac{3}{7}$ by $\frac{2}{3}$ maketh $\frac{2}{7}$, all theſe added togither, the totall is $25\frac{1}{3}$. The ſame will come to paſſe, if the whole numbers propounded be reduced to partes thus, $\frac{38}{7}$ and $\frac{14}{3}$, which being multiplied and therewithall reduced are $\frac{76}{3}$, and reduced to whole numbers are $25\frac{1}{3}$. Diuiſion likewiſe may ſometime be wrought ſeuerally. As if $5\frac{1}{3}$ be diuided by $2\frac{2}{3}$. Firſt 2 may be twiſe taken from 5, and there will remaine 1, that is $\frac{3}{3}$, which being added to $\frac{1}{3}$ maketh $\frac{4}{3}$, from whence yée may take $\frac{2}{3}$ twiſe alſo, wherefore the quotient is 2. But in greater numbers the eaſieſt way is by reduction, and ſo worke all ioyntly as $\frac{16}{3}$ diuided by $\frac{8}{3}$ is 2. So $\frac{5}{2}$ diuided by $\frac{7}{4}$ the quotient is $\frac{10}{7}$.

Chap. XIIII.

Comparing of numbers and the noting and numbering of reſpects.

HItherto of the abſolute part of Arithmeticke, the comparing part followeth, which declareth the compariſon of numbers, both in quantitie and qualitie. The former is either equall or vnequal, the compariſon in equall quantitie is but of one ſort, as 1 to 1, 2 to 2 &c. The compariſon of vnequall quantitie is either difference or reſpect. Difference is the compariſon how much one terme differeth from another, which is knowne by ſubduction. So the difference of 2 to 3, of 3 to 5, of 5 to 8, is 1, 2, 3, founde by ſubducting the leſſe from the greater. Reſpect is the compariſon how often one terme is contained in another, which is knowne by Diuiſion: as the reſpect being propounded, the termes contrariwiſe are founde out by multiplication. So the reſpect of 3 to 2 is ſo much and halfe, becauſe 3 containeth 2 once and halfe 2. So the reſpect of 5 to 3 is ſo much & two thirds. Of 4 to 2 is double. The quotients therfore expreſſing theſe reſpects, are $1\frac{1}{2}$,

$1\frac{1}{3}$ $\frac{2}{1}$, which are brought to their terms again, if the whole number be multiplied by the namer, and the numberer added thereunto for the former terme, and the namer alone set for the latter terme. But when respects must be nũbered, the former terme is alwayes noted aboue, the latter terme beneath, and their numbering is farre vnlike absolute numbering: for here multiplication is addition, and Diuision is Subduction. The addition of respectes is the multiplication of the former termes betwéene themselues, and the latter betwéene themselues, and it is called the cõpounding of respects. So the respect of $\frac{1}{1}$ added to the respect of $\frac{2}{1}$ is $\frac{2}{1}$, because the respect of equall quantitie increaseth nothing: so the respect of $\frac{3}{2}$ added to $\frac{7}{4}$ is the respect of $\frac{21}{8}$, so a Respect is said to be doubled and tripled, when the termes thereof twise or thrise set downe are multiplied. As the respect of $\frac{3}{2}$ is thus doubled $\frac{3}{2}\,\frac{3}{2}\,(\frac{9}{4}$, and thus tripled $\frac{3}{2}\,\frac{3}{2}\,\frac{3}{2}\,(\frac{27}{8}$. Therefore if ye termes of the respects be continued, the respect of the vtmost termes is compounded of all the middle respects: as in 1, 2, 3, 4, 5, the respct of 1 to 5 is compounded of all the rest betwéene thẽ, as here is séene $\frac{1}{2}\,\frac{2}{3}\,\frac{3}{4}\,\frac{4}{5}\,(\frac{24}{120}$ for reduction will shew that the respect of 24 to 120 is the respect of 1 to 5. Subduction of respectes is the diuision of their termes: as if the respect of $\frac{3}{2}$ be taken from the respect of $\frac{7}{4}$ there will be left the respect of $\frac{7}{6}$.

CHAP. XV.
The diuers ſorts of reſpectes.

COncerning the diuers ſortes of reſpectes, euerie reſpect is of greater inequalitie or of leſſe. The reſpect of greater inequalitie is named of the greater terme: the reſpect of the leſſe inequalitie hath this word vnder ſet before it. As the reſpect of 2 to 1 is called double, the reſpect of 1 to 2 is called vnderdouble. And either of theſe is vnmixt or mixt: that which is vnmixt is of one ſort alone, and the ſame ſingle or manifold. Single reſpect is when the greater terme conteineth the leſſe once and ſomwhat elſe, as reſpect particular and parting. Reſpect particular is when the one terme containeth the other once and one part beſides: if a ſecond part, a third part, a fourth part, it is called ſo much and halfe, ſo much and third, ſo much and fourth, &c. As $\frac{3}{2}$ $\frac{4}{3}$ $\frac{5}{4}$. Reſpect parting, is when one terme containeth another once, and ſome parts beſides: if two thirds, three foures, foure fiues, it is called ſo much and two thirds, ſo much and three foures, ſo much and foure fiues, as $\frac{5}{3}$, $\frac{7}{4}$ $\frac{9}{5}$. Manifold reſpect is when one terme iuſtly containeth the other oftentimes, if twiſe, thriſe, foure times, it is called a double reſpect, three-

fold,

fold, fourefold, as $\frac{2}{1}\ \frac{3}{1}\ \frac{4}{1}$. The mixt reſpect containeth diuers ſorts of reſpects, as manifold and particular manifold and parting. Reſpect manifold & particular, is whē one terme containeth another oftentimes and one part beſides: as double and halfe, thrée fold and third, foure fold and fourth, as here $\frac{5}{2}\ \frac{10}{3}\ \frac{17}{4}$. Reſpect manifold and parting is when one terme containeth another oftentimes and mo parts beſides, as reſpect double and two thirds, thrée fold, and thrée foures, foure folde and foure fiues, as here $\frac{8}{3}\ \frac{15}{4}\ \frac{24}{5}$. Thus are the diuers ſorts of reſpectes declared, and generally compriſed in two rules, the one of diuiſion for the finding out of the ſort of reſpect: the other of multiplication for the finding of the termes of the reſpect. And much more briefly may this Arithmetike be practiſed onely in the termes: as the bredth of Noahs Arke in reſpect of the height was as 5 to 3, which is ſo much and two thirds, the length in reſpect of the bredth as 6 to 1, which is ſix fold. The reſpect of a circle to a ſquare of the ſame diameter is as 11 to 14, that is, ſo much and thrée elevens: ſo that onely the termes doe repreſent the name of the ſort or maner of reſpect, and that in proportions, that is in the vſe of reſpects. And thus much of compariſon in the quantitie of numbers.

CHAP. XVI.
Arithmeticall proportion.

The comparison of numbers in qualitie is the equalitie of comparisons in quantitie, called Proportion, which is either Arithmeticall or Geometricall. Arithmeticall proportion is the equalitie of differences, as 4, 6, 8, 10. Where 2 is the difference directly, as 4 to 6, so 8 to 10. Also backwards, as 10 to 8, so 6 to 4, for still 2 is the difference. Againe interchangeably, as 4 to 8, so 6 to 10, where 4 is the difference. Arithmeticall proportion is in disioyned termes, or continued: that which is in disioyned termes hath two properties. The two middle termes added togither, are equall to the two vtmost added togither: and the ofcome of the middle exceedeth the ofcome of the vtmost as much as is the ofcome of the difference of the greatest aboue the middle, by the differēce of the same middle aboue the least. As 8 & 6 are 14, so 4 and 10 are 14. Againe the ofcome of 8 and 6 is 48, which exceedeth 40 the ofcome of 4 and 10, the vtmost by 8 the ofcome of 4 the difference of 10 aboue 6, by 2 the difference of the same 6 aboue 4 the least. The first propertie teacheth the inuention of the middle termes betweene two vtmost termes pro-

pounded: for any parts of the vtmoſt termes added togither are Arithmetically proportionall betwéene the vtmoſt as here may be ſéene betwéene 3 and 9,

		12	
3.	4.	8.	9
	5.	7	
	6.	6	

The ſecond propertie requireth that the termes be vtmoſt and middle not onely in order, but alſo in quantitie. As 8, 6, 10, 8, here 60 the oſcome of the middle, doth not excéede 64 the oſcome of the vtmoſt. Theſe are the two properties of Arithmeticall proportion in diſioyned termes. Arithmeticall proportion in continued termes, hath two properties likewiſe ariſing of the former. The middle is halfe of both vtmoſt termes added togither. And the oſcome of the middle terme excéedeth the oſcome of the two vtmoſt, as much as is the oſcome of the differences. As in 4. 7. 10. for 4 and 10 are 14, whoſe halfe is 7. Againe 49 the oſcome of 7 multiplied by it ſelfe doth excéed 40 the oſcome of 4 and 10 by 9 the oſcome of 3 & 3 the differences. The firſt propertie ſheweth the inuention of the middle terme by taking halfe of the two numbers propounded. The continuing of the termes in continued Arithmeticall proportion may procéede infinitly and is called Arithmetical progreſsion: wherein is a double inuention, firſt of any terme that yée liſt, then of the ſumme. The inuention of any terme is thus, If an vnitie be taken from the name of the deſired

terme, and the ofcome of the rest by the difference be added to the first terme, the whole will be the desired terme: as in a progression by 3 from an vnitie, if the twelft terme be sought for, here 12 is the name of the terme, take 1 from 12 the rest 11 being multiplied by 3 the difference maketh 33, thereto adde 1 the first terme, the whole is 34 the twelft terme in a progression by 3 from an vnitie thus, 1. 4. 7. 10. 13. 16. 19. 22. 25. 28. 31. 34. The inuention of the sum is thus. The totall of both vtmost termes multiplied by halfe, the name of the last terme is the summe. As in the former progression: adde 1 to 34, the total is 35, which multiplied by 6 halfe the name of ẏ last terme maketh 210 the whole sum. Halfe the totall of both vtmost termes multiplied by ẏ name of the last terme is ẏ whole sum likewise, as $17\frac{1}{2}$ multiplied by 12. But to illustrate ẏ matter by an example of practise, let vs admit ẏ a town is besieged 42 dayes with such a force as spent 300 ℓ the first day, and the armie dayly increasing the charges likewise increased 4 ℓ euerie day: how much was the whole charges? 1 take from ẏ last terme ẏ rest is 41, which multiplied by 4 the difference, & added to ẏ first terme, yée haue 464 for ẏ last terme, whereunto adde the first terme, it is 764, that multiplied by 21 halfe the name of the last terme, yée haue 16044 ℓ, which is the totall charges of the siege continuing 42 dayes.

Chap. XVII.
Geometricall proportion and the golden rule thereof.

Hitherto of Arithmeticall proportion. Geometricall proportion is the equality of respects, and it is properly called proportion of numbers, and the numbers called proportionals, as 2. 4. 3. 6. both directly, as 2 to 4, so 3 to 6, and backwards, as 6 to 3, so 4 to 2, and interchangeably both wayes, as 2 to 3, so 4 to 6, againe as 6 to 4, so 3 to 2. Here is also a double propertie, for the greatest and the least are greater then the rest. And the ofcome of the middle is equall to the ofcome of the utmost. The first propertie is separable, but the last propertie is unseparable from proportion, & therefore hath wonderfull use, insomuch that it is called the golden rule: for by this proportion are the middle termes found out, when the utmost are propounded: and the fourth is found out when three are propounded, for the divider of the ofcome of ÿ utmost and the quotient are the middle proportionall termes: as 2 & 6 the utmost propounded their ofcome is 12, whose divider 4 & quotient 3 are the middle proportionals. Again three termes being propounded, if the first divide ÿ ofcome

of the two other, the quotient will be ẏ fourth proportionall, as 2, 4, 3 being propounded, if 2 diuide 12 the ofcome of 4 and 3, the quotient 6 is the fourth proportional. And here the proportionall couples being compound betweene themselues for more ease are to be reduced to their vncompounds. And often times the fourth proportionall terme may be found out only by the definition of the proportion, especially in respects particular and manifold, as 2 to 3, so foure to what? or, as 2 to 6, so 3 to what? in the former we see that the second to the first is so much and halfe, in the later thrée fold, & therefore the fourth to the third ought to haue like respect, as are 6 and 9. Furthermore Geometricall proportion, as Arithmeticall is either disioyned or continued. Proportion disioyned is either single or in foure termes, or manifold in moe then foure. In the first sort if the termes be directly proportionall they must stand in such order that the first be in the first place, and the rest euery one in his proper place: and also that the first be of like kind to the third, and the second to the fourth, for so they are more cōmodiously placed interchangeably togither, and that the thirde terme allwayes make the question. Therefore when the question is set forth disorderly, the termes first must be brought into right order: as if it be demanded what 12 yards of cloath will cost when as 4 yards cost

5 s,

5 s̄, which must thus be brought into due order, 4 yards cost 5 s̄, therfore 12 will cost 15 s̄, so are the first terme and the thirde of like kind, to wit, of yards: also the second and the fourth, to wit, of shillings standing as you see here,

4.	5
12.	15

The third terme making the question which is answered by finding out the fourth proportionall terme. Sometimes three termes are propounded darkely: as if it be demanded how much $\frac{2}{3}$ are of 24. Here yee haue three termes 3. 2. 24. wherefore the fourth is 16 thus, $\frac{2}{3}\ \frac{16}{24}$.

Chap. XVIII.

Examples requiring an absolute numbering in whole numbers before proportion.

Now before the principall proportion sometime there goeth an absolute numbering, sometime another proportion. The absolute numbering is in whole numbers or in parts. In whole numbers take first this example of Addition, an 100 tunne of wine are bought in Burdeaux for 1050 li, and there was paid for custome of it 10 li, for the fraight to Plimoth 133 li, for the custome and impost there, 265 li, let the expenses of the factor be 20 li, now the marchant would gaine

180 l, how may he sell 40 tunne thereof? adde all the charges togither, it is 1658 l, then say an 100 tunne are worth 1658 l, therefore 40 tunne are worth 663 l $\frac{1}{5}$. An example of subduction, a marchant bought certaine wares for 8 l, sold the same for 10 l, how much should he haue gotten if he had bestowed an 100 l that way? First yee shall see by subducting the principall from the gaine, that the gaine of 8 l is 2 l, then conclude that seeing 8 l gaineth 2 l, an hundreth will gaine 25. Likewise when wares are bought for 8 l, and sold for seuen pound, the losse in such a bargaine vpon an 100 l will come to 12 $\frac{1}{2}$ thus: eight looseth 1, therfore 100 looseth 12 $\frac{1}{2}$. An example of multiplication. If things of diuers kinds be confounded togethers, first they shalbe reduced to one kinde: as 15 l in a certaine time gaineth 48 s, what will 20 nobles gaine? conuert the 15 l into 45 nobles by multiplying with 3 the nobles of a pound, then conclude the proportion, 45 nobles gaine 48 s, therefore 20 nobles will gaine 21 $\frac{1}{3}$ s. An example of multiplication & addition. A Vintner hath bought a tunne of wine for 17 l, and would gaine foure pound; how must he sell a gallon? Here first the tun must be reduced to 240 gallons (for so many it containeth) & the price 17 l be added with the gaine 4 l which make 21, then say 240 gallons are to be sold for 21 l, therefore one

gallon for $\frac{21}{240}$ of a pound, which is 21 d, seeing a pound containeth 240 pence. Change 5 ℓ into shillings, groates and pence; euerie one of equall number, how many must there be of ech sort? bring the pounds into the least value pence by multiplication, & yée haue 1200, whereof 12 are a shilling, 4 a grote, 1 a penie: adde these values together, they make 17, whereupon the proportion is concluded: 17 containeth euerie sort once, therefore 1200 containeth them 70 $\frac{10}{17}$ times. All which sorts being added together, ye shall haue the former summe. The example folowing is somewhat vnlike, where the two first termes are contained in the forme of the respect: as, to what number is 18 double and thrée foures? which forme of respect is thus written $2\frac{3}{4}$. Multiplie the quotient by the namer, thereto adde the numberer, ye haue 11 for the former terme, the namer 4 is the latter, thus then haue ye the thrée termes 11. 4. 18 whereby is found out the fourth 6 $\frac{6}{11}$. An example of multiplication and subduction. A foote man which goeth 23 miles in a day, is sent from Plimmouth to London with letters: an horseman which rideth 40 miles a day is sent away thrée dayes after to call him againe: when shall he ouertake the footeman? Here you see by subduction, that the horseman trauaileth euerie day so much as the footman and 17 miles more: and by multiplication

that the footeman hath gone 69 miles before the horseman begin his iourney, thẽ conclude that seeing the horseman increaseth vpon the footeman 17 miles in 1 day, he will increase vpon him 69 miles in $4\frac{1}{17}$ of a day. Diuision also may be vsed before proportiõ, as if 1000 bushels of wheat are worth 280 l., what are 40 quarters worth? Diuide the 1000 bushels by 8, so they are reduced to 125 quarters, and then euerie other terme being of like kinde, seeing that 125 quarters are worth 280 l., 40 quarters are worth $89\frac{2}{7}$ l. Let these suffice for examples of absolute numbering in whole numbers before proportion.

Chap. XIX.

Examples requiring absolute numbering in broken numbers before proportion.

THere follow examples of absolute numbering in the parts & peeces of a number before proportion. A man hath paide $\frac{1}{3}$ of his debt, then $\frac{1}{4}$, then $\frac{1}{8}$, and there remaineth 9 l., how much was the whole debt? the parts added togither are $\frac{17}{24}$, therefore the rest is $\frac{7}{24}$. Wherefore the proportion is concluded thus, 7 are worth 9, therefore 24 are worth $30\frac{6}{7}$ l. But here the numbering of broken numbers in the termes of

proportion haue more labour, as in concluding euerie payment seuerally which is more easily done by ioyning the whole numbers with the peeces, setting $\frac{216}{7}$ for $30\frac{6}{7}$, as 24 are worth $\frac{216}{7}$, therefore 8 payed first are worth $\frac{72}{7}$, $10\frac{2}{7}$, 6 payed next are worth $\frac{54}{7}$, $7\frac{5}{7}$, 3 payed last are worth $\frac{27}{7}$, $3\frac{6}{7}$.

CHAP. XX.

Examples requiring a proportionall numbering before the principall proportion.

TOuching examples of another proportion going before the principall proportion, first is that which hath an Arithmeticall proportion going before. As a certaine pioner tooke to task for 24 li the digging of a pit 120 foote deepe, whereof euerie foote exceeded other in labour so much as was the labour of the first foote: hauing digged 70 foote he was taken away to serue his Prince, wherefore he demaundeth his hire according to the worke done: how much must he haue? Seeing the labour increaseth by the difference of 1 in Arithmeticall progression, first seeke out the summe thereof. The last terme is 120, therefore the

whole is 7260, and the ſumme of the 70 terme is 2485: wherefore conclude that ſeeing 7260 haue 24 l, 2485 muſt haue 8 l $\frac{26}{171}$, that is 3 s, and $\frac{84}{171}$ of a penie. In the examples following a geometricall proportion goeth before with ſome abſolute numbering. A poſt goeth from Plimmouth to London in 5 dayes, another commeth from London to Plimmouth more ſpeedily in 4 dayes, admit that they begin their iourney both on Monday in the morning, when and where ſhall they meete? Set downe the former proportions thus: the firſt endeth his iourney in 5 dayes, therefore in 1 day he will ende $\frac{1}{5}$ of the iourney: the ſecond endeth his iourney in 4 dayes, therefore in 1 day $\frac{1}{4}$: theſe partes added together, are $\frac{9}{20}$ of the iourney. Whereupon conclude, ſeeing $\frac{9}{20}$ is gone in 1 day, $\frac{20}{20}$, that is 1 is gone in $\frac{20}{9}$ that is $2\frac{2}{9}$ of a day. This is the time of their meeting, to wit, Wedneſday at 8 of the clocke and 40 minutes in the fore noone. Then ſay, the firſt in 5 dayes goeth 1, therefore in $\frac{20}{9}$ dayes he will go $\frac{4}{9}$ of the iourney, which is the place of meeting: and then the ſecond hath gone the reſt of the iourney, to wit $\frac{5}{9}$. There are two artificers, of the which the firſt wil end the worke in 20 dayes, together with the ſecond he will end it in 14 dayes, in how many dayes will the ſeconde end it alone? The firſt in 20 dayes endeth 1, therefore in 14 dayes he will ende $\frac{7}{10}$, then the

the ſeconde in 14 dayes will ende $\frac{3}{10}$, nowe conclude, that ſeeing $\frac{3}{10}$ are done in 14 dayes 1 is done in $46\frac{2}{3}$ of a day.

A generall hath prouiſion inough to ſerue onely his horſe-men 4 yeares, onely the pike-men 3 yeares, the halbardes onely 2 yeares, the ſhot onely $\frac{3}{4}$ of a yeare: but how long will it be ſufficient for them all? The horſe-men ſpende all the prouiſion in foure yeares, therefore in 1 yeare they will ſpend $\frac{1}{4}$ of it, the pike-men $\frac{1}{3}$, the halbardes $\frac{1}{2}$, the ſhot $\frac{4}{3}$, theſe beeing added are in all $\frac{29}{12}$. Wherefore conclude of all the armie, as they ſpende $\frac{29}{12}$ of the prouiſion in 1 yeare, ſo will they ſpende 1, that is the whole prouiſion in $\frac{12}{29}$ of a yeare, that is 151 dayes and $\frac{1}{29}$ of a day. A ceſterne hath 3 cockes, of the which the firſt will emptie the ceſterne in $\frac{1}{4}$ of an houre, the ſecond in $\frac{1}{2}$ of an houre, the third in 1 houre: in what time will they emptie the ceſterne if they runne all three at once? ſay $\frac{1}{4}$ of an houre emptieth the ceſterne once, therefore 1 houre emptieth it foure times: alſo $\frac{1}{2}$ emptieth it twiſe, 1 houre once: adde theſe turnes together, they are 7: then conclude, the ceſterne is emptied 7 times in one hower, therefore once in the ſeuenth part of an houre. The termes are thus,

7	1
1	$\frac{1}{7}$

The firſt artificer would ende the worke in

30 dayes, the ſecond in 40, both togither with a third would end it in 15 dayes: in how manie dayes would the third end it alone? the firſt in 30 dayes would ende the whole worke, therefore in 15 dayes he will ende $\frac{1}{2}$, the ſecond in 40 dayes endeth the worke, therefore in 15 dayes he will finiſh $\frac{3}{8}$: then adde $\frac{1}{2}$ $\frac{3}{8}$, yée haue $\frac{7}{8}$, which being ſubducted from the whole there will remaine $\frac{1}{8}$, which the third will do in theſe 15 dayes, therefore he will do $\frac{8}{8}$ that is the whole in 120 dayes. The termes ſtand thus,

$$\begin{array}{ccc} 120 & \frac{1}{8} & 15 \\ & 1 & 120 \end{array}$$

Of this ſort alſo are deuiſed diuers other queſtions, which neuer or ſeldome happen in practiſe, and therefore haue ſcarce anie vſe or fruite at all. Such are theſe that follow. Two men hauing diuers ſummes of money the firſt ſaith to the ſecond, if thou giue me 2 s of thy money I ſhall haue triple the value of thine: Nay (quoth the ſecond) it were more méete that our ſums ſhould be made equall, and ſo would it be if thou giue me 3 ſhillings of thine, how much had each of them? The firſt hath 1 number of ſhillings, whereunto adde 2 vnities of the ſecond, ẏ whole is 1 number and 2 vnities, which being thrée fold to the ſecond, the ſecond hath $\frac{1}{3}$, and $\frac{2}{3}$, adde to the ſecond his owne 2 vnities, the whole will be $\frac{1}{3}$ of a number and $2\frac{2}{3}$ vnities. Now to take 3 from the firſt and giue them to the ſecond, is as much as to take nothing from the firſt,

and giue 6 to the second, then will the second haue $\frac{1}{3}$ numbers and $8\frac{2}{3}$ vnities, which are equall to one number of the first: therefore $8\frac{2}{3}$ vnities are equall to $\frac{2}{3}$ of a number, and consequently halfe so much more, to wit, 13 equal to $\frac{3}{3}$ that is 1 number: so much therefore had the first, whereto adde 2, he hath in all 15, then the second hauing a third hereof can haue but 5 left, giue him his 2, he hath 7, adde 3 more taken from the first, he shall haue ten equall to the residue of the first. The like will it be if yée beginne with the second, let the portion therefore of the second be 1 number, whereto if thrée vnities be added from the first, both are equall ech one hauing one number 3 vnities, restore to the first his 3 vnities, he hath 1 and 6. Now to take 2 from the seconde and giue them to the first, is all one as if yée take nothing from the second & adde 8 to the first, that is 2 and the triple of 2, that the one may be thrée fold to the other. So hath the first one number 14 vnities, this containeth one number of the second thrée times, therefore 1 number is equall to 7 &c. Two men haue 30 s so diuided, that the third part of the first and halfe of the second summe will make 13, how much had ech man? take $\frac{1}{2}$ of both portions, to wit 15, this containeth $\frac{1}{3}$ of the first and $\frac{1}{2}$ of the second, to wit 13 and 2 besides, that is $\frac{1}{6}$ of the first, as appeareth if yée take $\frac{1}{3}$ from $\frac{1}{2}$: wherefore conclude as 1 to 2, so 6 to 12, which

is the portion of the first, then had the second 18: now $\frac{1}{3}$ of 12 and $\frac{1}{2}$ of 18 are 13. The like may be concluded if ye begin with the other first. There is a band of souldiers (quoth one) armed with three sorts of weapon, pikes, halbards and shot, the halbards and shot are double to the pikes, the pikes and shot are eightfold to the halberds, the shot alone are 50 more then the two other: how many were of euerie sort? the halberds and shot to the pikes are as 2 to 1, the pikes and shot to the halberds are as 8 to 1. Adde the termes of the former respect, yée haue 3. add also the termes of the latter respect, yée haue 9: diuide 9 by 3 the quotient is 3. So yée sée that 3 of the latter respect is equall to 1 of the first respect, and therefore 1 representeth the halberds, 3 the pikes, 5 the shot: now the shot being 50 aboue the two other, and 5 being 1 more then 1 and 3, we sée that 1 standeth for 50 halberds, 3 for 150 pikes, 5 for 250 shot: the whole is 450. One man hath fiue debters whose seuerall summes though he haue forgotten, yet he findeth by his reckonings that the debt of the first & the second was 25 li, of the second and the third 39 li, of the third and the fourth 54 li, of the fourth and the fift 70, of the fift and the first 50, how much must be demaunded of ech debter? adde all these summes togither yée haue 238, which containeth the whole debt twise, therefore the debt

is

is 119 R, from whence take the first and the second, the third and the fourth, that is 25 and 54, the rest 40 is the fift mans debt, which taken from 50 there remaineth 10 the debt of the first, and so yee see the debt of the second to be 15, of the third 24, of the fourth 30. But of these more then enough, neither is there anie need of proportion in this last example, seeing it is ended onely by addition and subduction.

CHAP. XXI.
Backward proportion.

Hitherto of single proportion proceeding directly: but many times it is returned backewards, to wit, when as the first terme of the first respect is to the first terme of the second respect, so the secōd terme of the second respect is to yͤ second terme of the first respect: where the respect must be taken onely of termes yͭ are of like kinde, as of measures betweene themselues, of prices betweene themselues, of times betweene themselues, and values betweene themselues. And then the third terme of the backward proportion is sought for, therefore the ofcome of the two vtmost being diuided by that middle which is propounded will yeeld the other middle in the quotient. This proportion must be vsed

as often as the values being increased the things must be decreased: or the values being decreased the thinges must be increased, which may be more easily understood by the things numbered, then by the nature of the numbers: as when wheate is 2 s the bushell Winchester, the penie white loafe wayeth 60 ounces: therefore wheate 4 shillings the bushel, the penie white loafe wayeth 30 ounces: that ye returning backward may be seene, the example will be thus,

2	60
4	30

The same proportion by turning the order of the termes is thus concluded. As 30 to 2, so 60 to 4. Where three termes being propounded that which is brought forth in the last place shall be first, and that which is in the first place shall be last in the backward proportion, and so the termes of one kinde make the respect: as the first price with the second maketh the first respect, the second waight with the first maketh the second respect, as examples make it manifest. As a band of 4000 souldiers being besieged hath provision sufficient for eight moneths, for how manie souldiers will it suffice 12 moneths? multiply 4000 by 8, divide by 12, the quotient is $2666\frac{2}{3}$ thus,

8	4000
12	$2666\frac{2}{3}$

Three marchants have gotten 30 l, the first in 6 moneths of 60 l, the secod in 7 moneths, the thirde in 5 moneths: of what stocke had

ech

ech of the two latter this gaine? In this example and all like, one terme 30 being often repeated must be reiected, and be considered for the common gaine: 6 moneths occupie 60 li. therfore 7 will occupie lesse, & 5 will occupie more, as

6—60
7—51 3/7
5—72

CHAP. XXII.

Proportion compounded by addition.

Hitherto of single proportion in foure termes, now followeth manifold proportiō, wherein are used more termes then foure, and that either by compounding or continuing. The compounding of the termes is of the first sort, or of the second: of the first sort are addition and mixture. Addition is when the termes of the respect propounded are added togither, and it is of three fashions. The first is the adding of one former terme with the following terme to the same following terme: as 2 to 3, so 4 to 6, therfore as 5 to 3, so 10 to 6. What number is that to which if yee adde 3/5 of it self, the whole will be 24? here 3 is the former terme, 5 the terme following: adde both, they are 8 to 5 the terme following, whereupon conclude, as 8 to 5, so 24 to 15. The second addition is of many former termes to one terme following, or of one former terme to

manie termes following: as 2 to 4, ſo 3 to 6, & as 8 to 4, ſo 12 to 6: therefore 10 to 4, as 15 to 6. Againe, as 6 to 2, ſo 9 to 3, and as 6 to 4 ſo 9 to 6, therefore as 6 to 2 and 4, ſo 9 to 3 and 6. This ſecond faſhion of addition is verie ſeldome, and that only in ſchoole demonſtrations. The third faſhion is the addition of all the former termes to all the termes following: wherein the termes following are propounded altogether in one, but the former termes are propounded ſeverally, and then being added together are ſeverally concluded al the termes following. This faſhion of compounding hath ſuch great uſe in ſocietie and dealing together, that it is called the rule of fellowſhip, but with a more ample name may it be termed the rule of Equitie and Juſtice, as the examples following do teach. There are two partners, of the which the firſt did put in 12 ł, the ſecond 9 ł, whereby they haue gottē 14 ł, how much is ech mās part? ad the former terms together ẏ queſtion is thus anſwered, 21 get 14, therefore 12 get 8, 9 get 6

21 — 14	And contrariwiſe	14 — 21
12 — 8	the principall is	8 — 12
9 — 6	concluded,	6 — 9

Here and afterwards the firſt terme is propoūded in effect & not in very dééd. There are thrée owners of one ſhip, the firſt hath $\frac{1}{2}$, the ſecond $\frac{1}{3}$, the third $\frac{1}{6}$, the ſhip hath gotten 240 ł how much is euerie mans ſhare? adde the former

mer termes, to wit, the parts, which make 6 for the whole ship, and conclude thus

6	240
3	120
2	80
1	40

264 li. are to be parted to three persons, in such wise that the first haue foure times so much as the second, and the second thrise so much as the third, how much is ech mans part? Here beginne with the last, if the third haue 1, the second will haue 3, the third 12, which being added conclude, 16 haue 264, therefore 12 will haue 198, and 3—$49\frac{1}{2}$, and 1—$16\frac{1}{2}$. The examples following are more excellent, in the which a iudge cannot giue iudgement vprightly without this rule of equitie. A certaine debter oweth to one man 168 li., to another 120, to the third 72, to the fourth 48: all the debters goods amount but to 272 li., wherefore there wanteth 134 pound, to satisfie all. Therefore to giue euerie man according to ẏ respect of the goods and debts, yee shall adde the former termes, and conclude 406 haue 272, therefore

168	112
120	80
72	48
48	32

The patrimony of 348 pound is to be diuided to fiue brethren in such manner that the first haue $\frac{1}{2}$, the seconde $\frac{1}{3}$, the thirde $\frac{1}{4}$, the fourth $\frac{1}{5}$, the fift $\frac{1}{6}$. This cannot be done as it is propounded because the partes doe exceede the whole: therefore the least diuidable of the partes must bee found out, and the partes

therein namelike to the partes propounded: the leaſt diuidable therefore of the namers is 60, whoſe parts namelike to the parts propounded are 30. 20. 15. 12. 10, which partes added for the former terme make 87, then conclude

87 haue	348
30	120
20	80
15	60
12	48
10	40

414 l' muſt be diuided to 4 perſons in ſuch wiſe that the firſt haue $\frac{1}{2}$ and 9 l', the ſecond $\frac{1}{3}$ and 7 l', the third $\frac{1}{4}$ and 8 l', the fourth $\frac{2}{5}$ lacking 6 l'. Here the leaſt diuidable of the namers being found, and the namelike parts to the parts propounded, the whole numbers that are aboue the parts, to wit 9, 7, 8 muſt be taken from the whole ſum 414, but the whole number vnder the part to wit 6 muſt be added, ſo the ſumme will be 396: afterward the fourth proportionall termes being foũd out, theſe whole numbers muſt be added and ſubducted according to ỹ manner of the example, which is thus ſet forth.

89 haue	396
30	$142\frac{45}{89}$
20	$95\frac{88}{89}$
15	$74\frac{66}{89}$
24	$100\frac{70}{89}$

Againſt this threefold compounding by addition, may be ſet a threefold diſioyning by ſubduction. The firſt is the ſubduction of the differẽce of the former terme from the terme following. As 7 to 3, ſo 14 to 6, therefore as 4 to 3, ſo 8 to 6. The ſecond is the ſubduction of any like number: as 12 to 2, ſo 18 to 3, therfore as 8 to 2, ſo 12 to 3. Againe as 4 to 2, ſo 6

to

to 3, for in both is taken away under so much and halfe. The third is the subduction of the difference of the former terme from the former, to the difference of the latter terme from the latter: as 12 to 9, so 8 to 6, therefore as 12 to 9, so 4 to 3. This last subduction is much used in schoole demonstrations in this wise. If that which is taken away be to that which is taken away, as the whole is to the whole, then the rest to the rest shall be as the whole is to the whole. There is also a returning backe, which is subduction of the former terme to his difference aboue the terme following: as 6 to 4, so 3 to 2, therefore as 6 to 2, so 3 to 1. And thus much of compounding by addition with the opposite subduction.

Chap. XXIII.
Mixture.

Mixture is the mingling of diuers sorts, whereof a meane is tempered: as in diuers kinds of graine, liquor, mettall, prices, waights, measures, and in all such things as may be mingled and tempered. This mixture of it selfe is no proportion, but oftentimes it useth a proportion, yea the former addition of proportion. And generally in mixture sometime the meane is sought for, sometime it is propounded. The

first sort is when the vtmost termes being propounded are added togethers and diuided by the number of thēselues to finde the mean: as if there be two vtmost termes the diuision must be by 2: if thrée then by 3, and so forth. As a bushel of barley mault of 17 groats must be mingled with a bushell of oten mault of 7 groats, yée shal adde 17 & 7, & diuide the whole 24 by 2, the quotient 12 is the meane, shewing the price of the mixed corne. But if with both of these bushels there must be mixed a bushell of wheaten mault of 22 groats, adde the thrée prices, diuide the whole 46 by 3, the quotient $15\frac{1}{3}$ sheweth the price of the fourth mixed graine. The second sort of mixture, is when the meane propounded is made equall by the interchangeable differences of the vnequall vtmost termes from the meane. As two sorts of wine, whereof the one is worth 11 d the quart, the other 5 d, must be mixt togither, so as a quart thereof may be worth 7 d. Here 2 & 4 are the interchangeable differences of the vtmost termes 11 and 5 from the meane 7, which differences signifie, that if 2 quarts be taken of the first sort, 4 must be taken of the second sort: therefore if 6 quartes be mixed, the mixture will be finished without any proportion, as here,

11.		2
	7	
5.		4

The cause of this mixture ariseth of the cōmon rules of multiplicatiō: for if yée multiplie 7 by 6, yée make 42; againe if yée

mul-

multiply 7 by 2 & 4 the péeces of 6, yée make 14 & 28, which is equal to 42, (for it is all one to multiply by ÿ whole, & by the parts) then if by the same 2 & 4 yée multiply ÿ whole 7, now diminished, now increased with the péeces interchangeably, to wit 11 and 5, yée make 22 and 20 equal to 42, as appeareth

7	7	7	11	5
6	2	4	2	4
42	14	28	22	20

But 2 quarts of the first sort of wine multiplied by 11 the price make 22, & 4 quarts of the second sort multiplied by 5 the price make 20, then 22 & 20 are 42. Againe 2 quarts multiplied by 7, & 4 quarts by 7 make 14 & 28 that is 42, therefore as much as the quartes are worth unmixt, so much are they worth being mixt. Thus we sée ÿ mixture is wrought without proportiõ, neither doth it alwaies require addition of proportion, when it useth proportion. As if a goldsmith haue 100 ounces of gold of the finesse of 17 carrets the ounce, and another masse of 24 carrets fine, to mix hereof gold of 22 carrets the ounce in finesse, how much of the second sort shalbe added to the first? the knitting of the interchangable differẽces wilbe thus,

$$22\begin{cases}17. & 2\\ 24. & 5\end{cases}$$

Wherfore conclude ÿ séeing 2 of ÿ first sort will haue 5 of the second, 100 will haue 250, whereunto adde the 100 of the first sort, & yée haue 350 the waight of the mixed gold. But the question of mixture is seldome without the addition of proportion, as in the first

example, if 60 quartes were to be tempered of those two sorts of wine, then the mixture being made, and the proportion added, ye shall conclude that as 6 haue 60, so 2 haue 20, & 4 haue 40: the whole question will be thus.

7 { 11. 2 / 5. 4 } 6—60 { 2—20 / 4—40 }

The like reason of mixture is there, where there are more termes thē two, for alwayes two of the vtmost termes must be compared to the meane, and the difference interchangeablie knit to the vtmost termes, and collected by addition. Now the knitting of termes may be diuers wayes, but the readiest way is to knit two and two together, if the multitude of the lesse termes be equall to the multitude of the greater: otherwise one terme must be knit to moe then one. As a goldsmith hath foure sorts of siluer of sundrie finenesse, to wit, of 12 ounces, 11 ounces, 8 ounces and 5 ounces fine, & would mix it so as that it might be of 9 ounces fine: the whole example will be thus.

9	12.	4
	11.	1
	8.	2
	5.	3

Here if onely 10 pound of mixed siluer be sought for, there must be taken 4 of the first sort, 1 of the second, 2 of the third, and 3 of the fourth: but if there must be mingled either more or lesse then 10 pound, the proportion will resolue the question: so then in this example

ample the multitude of differences is equall on both sides, in the example following it is otherwise, for one and the same terme is thrise knit. A goldsmith hath foure sorts of gold, the best of 24 carrets, the second of 20, the third of 19, the fourth of 17 carrets fine, & would of all foure sorts make a scepter of 72 ounces, worth 21 carrets fine: the termes being set downe in order, and the differences interchangeablie lincked togither, the whole question will stand thus.

	24.	4. 2. 1		7 — $31\frac{1}{2}$
	20.	3		3 — $13\frac{1}{2}$
21	19.	3	16 — 72.	3 — $13\frac{1}{2}$
	17.	3		3 — $13\frac{1}{2}$

Chap. XXIIII.

Compounded proportion by multiplication.

The first sort of compounding termes by addition, and contrariwise of deducting them in mixture is thus declared. There followeth the seconde sort which is made by multiplying ẏ termes, when for two couple of single termes their two ofcomes are taken. But sometime multiplication is alone, sometime addition is used with it. Concerning multiplication of the termes alone, these are examples, 6 l' in

7 moneths get 2 li, what will 8 li get in 3 moneths? ẏ terms wil be thus,

6. 7. 42—2
8. 3. 24—$1\frac{1}{7}$

When a terme of the former respect shall happen to be equall to a terme of the latter, these equals being cast away, the rest will conclude the proportion, as the bushell of wheat 16 groates, 20 ounces of bread for 1 d, the bushell 20 groates, how 12 ounces of bread? to wit $\frac{3}{4}$ d thus,

16. 20. 320-1 or 16--1
20. 12. 240-$\frac{3}{4}$ 12--$\frac{3}{4}$

the equall termes 20 and 20 being omitted.

Hangings of $3\frac{1}{2}$ yards long, and 2 brode cost 5 li, therefore a carpet of the same sort 3 yardes long and $\frac{4}{5}$ broad will cost $1\frac{5}{7}$ thus,

$3\frac{1}{2}$. 2. 7—5
3. $\frac{4}{5}$. $\frac{12}{5}$—$1\frac{5}{7}$

300 souldiers in 6 moneths spend 1200 li, therefore 600 in 3 moneths will spend 1200 li likewise. Oftentimes by this maner of compounding, questions of the first sort of mixture are answered. As 14 bushels of wheat of 7 s the bushell, and 18 bushels of barley of 5 shillings the bushell mixed together, what is a bushell thereof worth? the ofcomes of 14 and 7, of 18 and 5 are 98, 90: and the totall of them added is 188, the numbers of the things 14 and 18 are 32: then the proportion concludeth thus, 32 make 188, therefore 1 maketh $5\frac{7}{8}$, the example is so,

14. 7. 98 | 32—188
18. 5. 90 | 1—$5\frac{7}{8}$

Admit there be 54 bushels of wheat price

5 s,

5 s̄, 30 of rie, price 4 s̄, 23 of barely price 3 s̄, yée shall procéede as before, the example will be thus,

54.	5.	270	107--459
30.	4.	120	
23.	3.	69	1--4$\frac{31}{107}$

The like forme must be vsed, when the parts of diuers thinges are to be mixed, sauing that for these parts, yée shall take their least diuidable, and his partes namelike to them propounded. As brasse is 8 d̄ the pound, copper 6 d̄, tinne 4 d̄: and yée would make a mixture thereof, taking of the first $\frac{1}{2}$, of the second $\frac{1}{3}$, of the third $\frac{1}{4}$: the least diuidable of the parts is 12, and the namelike parts are 6 4. 3. wherefore the example will be thus,

6.	8.	48	13——84
4.	6.	24	
3.	4.	12	1——6$\frac{6}{13}$

Somtime this proportion is turned backward by multiplying the first and the fift terme betwéene themselues to make the first, and the thirde and the fourth to make the third. As if 2 acres of land are worth 6 li in 4 yeares, in how many yeares will 8 acres be worth 20 pound? The example,

2.	4.	6	40——4
8.		20	48——3$\frac{1}{3}$

Which may be thus turned,

3$\frac{1}{3}$——40
4 ——48

Chap. XXV.

Compounding by addition and multiplication both at once.

The compounding of addition & multiplication both at once first multiplieth the terms propounded, and then addeth their ofcomes togither. As of thrée marchants, the first laid in 44 l. 8 moneths, the second 32 l. 6 moneths, the third 24 l. 4 moneths, whereby they got 80 l, how much commeth to euerie mans part? multiplie the principal summes euery one by his time, and the compounded terme of the first will be 352, of the second 192, of the third 96 which being added together, say as 640 gaine 80 so

352	44
192	24
96	12

Somtimes there must be made manie ofcomes of the diuers sorts of principall sums, & at length added all into one. As foure marchants making a common stocke for two yeares, the first laid in 30 l, but 8 moneths after tooke away 10 l, again in the beginning of the twentieth moneth laid in 12 l, the second laide in 24 l at the beginning, but at the end of the first moneth tooke away 8, and in the beginning of the sixtéenth brought againe 7: the third laid in 20 l at the first, and after the seuenth moneths ende tooke away all, and at the

ſhe end of the ſeuentéenth moneth brought 16 againe: the fourth in the beginning of the ſeuēth moneth laid in 18 l', but 4 moneths after toke away 9 l': again in the beginning of the ſeuentéenth moneth added 15 l', the gaines of all theſe ſummes was 240 l', how much was euerie mans particular gaines? the reſoluing of this queſtion hath not ſo much labour as the queſtion it ſelfe, both which are thus ſet forth.

30.	8.	240	
20.	11.	220	620
32.	5.	160	
24.	6.	144	
16.	9.	144	495
23.	9.	207	
20.	7.	140	
0.	10.	0	252
16.	7.	112	
0.	6.	0	
8.	4.	72	318
9.	6.	54	
24.	8.	192	

Now let vs adde al theſe former compounded termes together, and conclude the principall proportion, 1685 gaine 240 l', therefore

620 gaine $88\frac{104}{337}$

495 ——— $70\frac{170}{337}$

252 ——— $35\frac{301}{337}$

313 ——— $45\frac{99}{337}$

Sometime the mixtures of the vnknowne meane happen to be of this ſort: when as ſome greater number is demaunded, as in that example of mettall, if for 13 pound there

were required 100, the example would be thus, 13 haue 100 therefore 6 will haue

—	—	$46\frac{2}{13}$
4	--	$30\frac{10}{13}$
3	--	$23\frac{1}{13}$

CHAP. XXVI.

Proportion that hath continued termes for the finding out of the termes in the reſpects propounded.

HItherto of manifold proportion, which hath compounded termes: now followeth a manifold proportiõ, whoſe terms are continued, which is when ſome terme of the former reſpect is continued in that which followeth: as the inuention of the leaſt numbers in the reſpects propounded, and equaſion For the firſt, when any reſpects are propoũded in the leaſt termes, if the leaſt proportionall numbers to the ſecond & the third, multiplie ouerthwartly the termes of the two firſt reſpects, the ofcomes will be the leaſt continued termes in the reſpectes propounded. Then if the leaſt proportional numbers to the terme laſt foũd, and to the former terme of the reſpect following multiply ouerthwartly, the one ẏ termes found out, the other all them that follow: the ofcomes ſhall be the leaſt continued termes in the reſpectes propounded, as here yee may ſee,

3.	4	6.	5
9.	12		10

For

For if yée take the leaſt proportional numbers to 4 and 6, yée haue 2 and 3, then multiplying ouerthwartly 4 and 3 by 3, yée make 12 and 9. Again multiplying 6 & 5 by 2, yée make 12 and 10 the leaſt continued termes in the reſpects propounded: for as 3 to 4, ſo 9 to 12, & as 6 to 5 ſo 12 to 10. This proportion is cõtinued onely in termes, but diſioyned in reſpectes, whereby yée may continue as many termes as yée liſt, whereof take another example,

1 . 2	3 . 4	16 . 5
3.	6.	8
6.	12.	16. 5

In this reſpect the proportionall to 8 the number laſt found out, and to 16 the former of the reſpect following are 1 and 2, which by ouerthwart multiplication make 6. 12. 16. 5. And this continuing of termes hath ſingular vſe in the rule of equitie, whereof we ſpake in compounding by addition. A certaine man making his teſtament, his wife being great with child, gaue to the child if it ſhould be a man child $\frac{2}{3}$ of his goods, ẏ reſt to his wife: but if it ſhould be a woman childe, he gaue to it $\frac{1}{4}$ of ẏ goods, and the reſt to his wife: ſhe brought forth a man child & a woman child, the inuentory of his goods was 180 ℔, how much was euerie ones part? firſt ſet downe the parts according to the minde of the teſtator $\frac{2}{3}$ $\frac{1}{3}$ for his ſonne and his wife, then $\frac{3}{4}$ $\frac{1}{4}$ for his wife & daughter, whereby we ſée that when the ſonne hath 2, the mother muſt haue 1, and when the mother hath 3, the daughter muſt

haue 1, which termes being continued will be thus,

2.	1	3.	1
6.	3.		1

Then adde the continued termes in one, and so conclude euerie part, 6 haue 108 10 haue 180, therefore

6 haue 108
3 —— 54
1 —— 18

Admit that a father bestow amongst his children 1560 ℔ in such wise that as often as the first hath 1, the second haue 4, and as often as the second hath 6, the third haue 5, and as often as the third hath 4, the fourth should haue 3, how much is euerie childs part? Here are thrée diuers respects in the least termes which we will set downe with their least cõtinued proportionall termes in this wise,

1.	4 | 6.	5 | 4.	3
3.	12.	10.	
6.	24	20.	15

Now these continued termes found, adde togither, the total is 65, wherefore conclude that if 65 haue 1560, 6 will haue

—— 144
24 —— 576
20 —— 480
15 —— 360

So if 3 d English be worth 2 sous Paris, and 4 sous Paris 5 sous Turnois: 6 d English shalbe worth 5 sous Turnois, as

3.	2	4.	5
6.		4	5

If yée would know further, as how manie sous Turnois make 30 d English, the proportion of 6 to 5 doth aunswere the question,

6 —— 5
30 —— 25

CHAP. XXVII.
Equation.

AND ſo is the finding out of the leaſt proportionall numbers in the reſpectes propounded. There remaineth equation, which is a continuing of two orders proportional in couples of numbers, therefore the vtmoſt are proportionall though the middle be taken away, neither is there required here a continuing of the reſpects, but only of the termes. Equation is orderly or vnorderly. Orderly equation is according to the ſame order of numbers, therefore the firſt numbers are proportional to the ſecond, and the ſecond proportionall to the third, as yee ſee here in three examples, which may be continued in one.

9.	6.	3	9.	6.	9	3.	6.	9
12.	8.	4	12.	8.	12	4.	8.	12

In this example the equation may be continued in eight proportions, and ſeuen proportions being omitted yee may ſay as 9 to 9, ſo 12 to 12. There is a fained example of a father that gaue 72 l. to his foure children, in ſuch proportion that the ſecond and the third ſhould haue ſeuen times ſo much as the firſt: the third and fourth fiue times ſo much as the ſecond: the fourth and firſt twiſe ſo much as the third; how much was the portion of eue-

rie child alone? Here for more perspicuitie we will represent the foure persons by these foure letters a. b. c. d, and by them also expresse the termes of respect in two orders interchangeably made equall thus,

b c.	c d.	d a
7 a.	5 b.	2 c

So are all the termes of the first order equall to all of the second. Wherefore according to that general rule, take like from like, the remaines wilbe like: which is agreeable also to the second subduction of proportiō. So then subduct a b c c equally from both sides, there will remaine d d equall to 6 a, 4 b, and therefore d equall to 3 a, 2 b, & consequently these 3 a, 2 b with a in ẏ last terme are equall to 2 c, then 2 a, b equall to c. Whereby in the second terme these equall values of d and c, to wit, 3 a 2 b and 2 a, b are equal to 5 b: herehence abate like frō both sides, there remaine 5 a equal to 2 b: then is ẏ respect of a to b, as 2 to 5, & so as appeareth in the first termes of b to c as 5 to 9, of c to d as 9 to 16, so these termes added together are 32, wherefore conclude if 32 haue 72 2 for the first will haue $4\frac{1}{2}$, 5 for ẏ second wil haue $11\frac{1}{4}$, 9 for the third $20\frac{1}{4}$, 16 for the fourth 36. But he that findeth anie vse of such kinde of questions may set forth this point more at large. Vnorderly equation is when as the first terme of the first order is to the second, so the second terme of the second order is to the third: as yée sée in thrée seue-

red examples which are not continued,

9.	8.	6.	9.	8.	9.	32.	16.	8
24.	18.	16.	16.	18.	16.	8.	4.	2

For here as 9 to 8, so 18 to 16; againe as 8 to 6, so 24 to 18. And so likewise turning the order in the rest of the examples. But it is hard in whole numbers to continue the termes of proportion, if they be made equall backwards. Howbeit they may be continued though the order be turned not onely backwards, but also to the contrary parts, as here yee see. 6. 3. 2. 1. 3. 4. 3. 1. 2
12. 24. 8. 6. 8. 24. 12. 8. 4

But this kinde of proportion is not verie vsuall, and thus disioyned proportion is generally described.

Chap. XXVIII.
Continued proportion.

Now it followeth that we speak of cõtinued proportiõ, which is whẽ the terme that followeth in the first respect, beginneth in the next respect as 3. 6. 12. wherfore the two properties of proportion are here in three termes, ẏ middle terme being twise repeated for two terms. Further cõtinued proportiõ is either single in three terms only, or else manifold in as many termes as yee list. And then

the respect of the first to the second is doubled in the third, made thrée fold in the fourth, and so in order still one lesse: as in 3. 6. 12, the respect of 3 to 12 is the respect of 3 to 6 doubled: in 3. 6. 12. 24, the respect of 3 to 24 is the respect of 3 to 6 made thrée fold: for the respect of the utmost termes is gathered of all the middle, as we haue séene before.

Chap. xxix.

The finding out of the termes in a continued proportion.

IN this progression yée sée that if the first diuide the second, it wil diuide all the rest. But especially two things are to be considered, the finding out both of the termes and also of the whole summe. The finding out of the termes is common to all respects, or proper onely to manifold respects. The first is thus, If two numbers of the respect propounded be multiplied ech by himselfe and the one by the other, there will be thrée ofcomes continued proportionally to the numbers propounded: then if ẏ ofcomes be multiplied by the former propounded number, againe the last ofcome by the latter propounded number, yée shal haue foure numbers proportionall to them propounded, and so farther as many as yée list.

As

As for example 2 and 3 propounded, whose respect is vnder somuch and halfe.

2. 3
4. 6. 9
8. 12. 18. 27
16. 24. 36. 54. 81

Hereof followeth the inuention of the least termes, if the terms propounded be the least, for the vtmost shalbe vncompound betwæne themselues, because they are the ofcomes of numbers vncompound betwæne themselues, or of numbers vncompound by numbers vncompound: but if they be proportionall without intermission, they shall be the least of the vtmost that are vncompound betwæne themselues: and contrariwise, as is manifest in the same example. Therefore if there be a continuing of the vtmost termes vncompound betwæne themselues, it shalbe the greatest. Of this inuention come two other inuentions. The first, if two numbers haue anie middle termes without intermission, anie other proportionall to the same shal haue iust so manie middle termes. As in 1. 2. 4. 8. 16. betwæne 1 and 8 are two like termes, to wit, 2 and 4. and so likewise betwæne 2 and 16, of the like respect are two like termes, to wit 4 and 8. The second is that vncompound numbers betwæne themselues haue so many middle termes without intermission as they haue to an vnitie, and that two num-

bers and an vnitie haue as many middle termes as haue the numbers propounded, as betwéene 8 and 27 are two middle termes 12 and 18, and so betwéene 27 and 1, to wit 9 and 3, and betwéene 8 and 1, to wit 2 and 4, as yée may further sée in the same example, if yée set before it an vnitie,

1

2. 3

4. 6. 9

8. 12. 18. 27

16. 24. 36. 54. 81

And this is ẏ finding out of ẏ terms cōmon to al respects, wherof followeth the other ẏ is proper only to manifold respects, which first findeth out euery terme in his order by multiplying the last terme by ẏ name of ẏ respect, as 1.2.4.8.16. Againe 1.3.9.27.81. multiply any terme in the first by 2, in the last by 3, & ye shal haue the terme following. Secōdly this particular inuention findeth out any terme that yee list thus, If the termes of Arithmeticall progression beginning at an vnitie be set against the termes of Geometricall progression, beginning at ẏ first nūber of multitude of the respect propounded, ẏ ofcome of two Geometricall termes shalbe a terme of his owne progression one more thē both Arithmeticall termes added together, which stand against ẏ termes multiplied, as in this double progression, 1. $\overset{1}{2}\cdot\overset{2}{4}\cdot\overset{3}{8}\cdot\overset{4}{16}\cdot\overset{5}{32}\cdot\overset{6}{64}\cdot\overset{7}{128}\cdot$ Now if yee would haue the

the tenth terme of this progressiō ad any two Arithmetical termes yͭ make 9, to wit 2 & 7, 3 & 6, or 4 and 5, and multiply yͤ Geometricall terms answerable to any of these couples, as 16 & 32 answerable to 4 and 5, the ofcome 512 is the tenth terme of this progression. So likewise in a threefold progression $1.\overset{1}{3}.\overset{2}{9}.\overset{3}{27}.\overset{4}{81}.\overset{5}{243}.\overset{6}{729}.$ If yée would haue yͤ thirtéenth terme because 6 added to it self maketh 12, multiply yͤ terme vnder it by it selfe, to wit 729, yͤ ofcome wilbe 531441 the thirtéenth terme. This kind of inuention is gathered of the Arithmeticall progressiō of the places, because the first number of multitude multiplying the second maketh the third, which thing is proper onely to the first place: for the second number multiplying yͤ third maketh not the fourth, but one farther, to wit the fift: the third number multiplying the fourth maketh not yͤ fift, but two farther, to wit the seuenth, & so of the rest. The cause of such vnequall increasing is the propertie of proportional numbers. Because looke how many continued middle nūbers are betwéene an vnity and the first number of the two that are multiplied, so manie are there betwéene the second of the numbers multiplied, and the number that yée séek for. For euerie progressiō of a manifold respect beginneth at an vnitie, because any number of multitude is manifold to an vnitie: and therfore the inuention of what terme yée list is as we haue said

only in manifold respects, as whose first term may be an unitie: but not in other manner of respects, for that their first terme cannot be an unitie. And this much of finding out the termes in a continued proportion.

CHAP. XXX.

The finding out of the summe in Geometricall progression.

THe inuention of the summe in Geometricall progression is thus. If the first terme be taken from the second and the last, as the rest of the second shalbe to the first, so shall the rest of the last be to all them before the last: wherefore if the fourth (which shall be to ẏ rest of the last, as the rest of the second to ẏ first) be added to the last, the totall will be the summe, as here yée sée,

2.	4.	8.	16
	2		14

Take 2 from 4 and 16: as 2 the remainer of the second is to the first 2, so 14 the remainer of the last is to 8. 4. 2, al the former numbers, for they are equall of both sides: therefore 2. 2. 14. 14 are proportionall. Now adde 14 to the last terme 16, the summe of the progression will be 30. So in a thrée folde respect,

1.	3.	27.	81
	2.		80

Take 1 from 3 and 81, then as the remainer 2 is to 1, so is the remainer 80 to all the former numbers, and therefore double, then

2. 1.

2.1.80.40, are proportionall. Now adde 40 to 81, the totall 121 is the summe of the progression. But now let vs sée an example or two. Where as Adam liued tenne generations, let vs admit that euerie generation one with another had 12 children, 6 males and 6 females multiplied by their families, which after this rate by reason of marriage must be 6 families of one: how manie were the children & nephewes of Adam before his death? Here we sée that for the families, the respect is sixe fold, then the first terme of Geometricall progression is 1, the first terme of Arithmeticall progression hath vnder it 6, the second 36, the third 216 thus, 1. $\frac{1}{6}$. $\frac{2}{36}$. $\frac{3}{216}$. the third multiplied by it selfe is 46656 for the firt terme, which being multiplied againe by 216 the third terme, yée haue for the ninth terme 10077696, which is the tenth terme of the Geometricall progression. Now take the first from the second and the last: as the remainer 5 is to 1, so is the remainer 10077695 to all the former: then the former vnder fiue fold is 2015539, which added to the last is 12093235. Now because a familie consisteth of two sorts of persons, man and wife, sonnes and daughters, for both sorts double this sum, so haue yée 24186470 for Adam and Eue with all their posteritie, which Adam might sée through the wonderfull blessing of God to be thus increased replenishing the earth. And

though that many might die before Adam, and some families had not so many as we suppose, yet it is likely that they had one with another more a great many, seeing that in sixe generations of Iacobs posteritie onely the male kind able to beare armour were 603550 persons, which after our supposition would be but 18662. It is written in the gospel that the seede which fell in good ground brought forth some an hundred fold, some sixtie, and some thirtie: whereupon I demaund to what quantitie a pint of graine will come in ten yeares increasing onely but ten fold yearely, because much perisheth without increase? Here because all the former termes are spent in sowing, the last terme alone answereth ye question, which is 1000000000, which being diuided by 512 the pintes of a quarter will be 1953125 quarters Winchester, euen seed enough (if it be of wheat) for more then twentie countries so great as England. The rule of waying manie things with few waights commeth of Geometricall progression. The pounds contained within the termes of a double and threefold progression are wayed with as many namelike waights: which is done sometime by adding one waight to another, sometime by taking away, and adding to the contrarie balance. So in a double respect all termes to 15 are wayed with 4 waights, to wit a waight of one

one pound, of two pounds, of foure pounds, of eight pounds: becauſe they are contained in theſe termes of progreſſion. 1. 2. 4. 8. So in a threefold reſpect all pounds to 40 are wayed with 4 waights ſignified by this progreſſion. 1.3.9.27. All pounds to 364 are wayed with ſixe waights ſignified by progreſſion thus, 1.3.9.27.81.243. and ſo proceeding infinitly in greater numbers.

FINIS.

And that thou hast
not heard nor [illegible]
of my Wife my [illegible]
[illegible] the meat of the
that thou didst
faret .

- Absolute, which cōsidereth of numbers, the
  - Noting their values which is
    - Single, as 1.2.3.4.5.6.7.8.9.0.
    - Increased, wherein are three degrees, once, ten times, an hundred times, which are againe likewise repeated by course, as 10. 120. 1230. 12340. 123450. 1234560. 1234567890.
  - Nūbering which is
    - Single, as
      - Addition.
      - Subduction.
    - Manifold, as
      - Multiplication.
      - Diuision, whereby a number either
        - Standeth whole, wh hath two differēces the
          - First
            - Odde, Euenly.
            - Euen Odly.
          - Second of
            - One alone
              - Vncompound.
              - Compound.
            - Mo togethers
              - Vncompound betweene themselues.
              - Compound betweene thēselues, where must be considered the
                - greatest cōmō diuider.
                - least cēmon diuidable.
        - Is broken into parts, which require
          - A peculiar kind of noting with two termes, the
            - Vpper is called the numberer.
            - Vnder is called the namer.
          - Reduction
            - Before numbering, of
              - Termes alone to their least proportionals.
              - whole nūbers, and parts, of
                - Whole numbers to parts.
                - Parts, to
                  - Whole numbers.
                  - Parts, to
                    - A couple proportionall parts.
                    - One equall.
            - Ioyned with numbering.
- Cōparing, which declareth the comparison of numbers in
  - Quantity, and that
    - Equall, but of one sort, as 2 to 2, 3 to 3.
    - Vnequal
      - Difference.
      - Respect, hath
        - A certain proper
          - Noting the
            - former aboue.
            - latter beneath.
          - Numbering, wherein multiplication is Addition, & Diuision, Subduction.
        - Two sorts, as respect of inequalitie the
          - Greater,
          - Lesse,
          - Either
            - Vnmixt
              - Single
                - Particular.
                - Parting.
              - Manifold.
            - Mixed
              - Manifold and particular.
              - Manifold and parting.
  - Qualitie, which is called proportion, and is
    - Arithmeticall, and it is
      - Disioyned.
      - Continued.
    - Geometrical, properly called the proportion of nūbers, & is
      - disioyned
        - Single in foure termes
          - Directly.
          - Backwards.
        - Manifold in moe termes, which is by
          - compoūding, and that
            - First
              - Additiō threefold, wherunto is opposed a threefold diuisiō.
              - Mixture of the meane
                - Sought for.
                - Propounded.
            - Second, multiplication of termes, sometime
              - alone.
              - with addition.
          - Continuing, which is
            - Inuention of the least numbers in the respects propounded.
            - Equation
              - Orderly.
              - Vnorderly.
      - Continued, and is
        - Single in three termes.
        - Manifold in mo termes, which is called progression, [illegible] considered the inuention of th[illegible]
          - Termes.
          - Summe[illegible]